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ROGERS–RAMANUJAN TYPE IDENTITIES RELATED TO MODULI $4k$ AND $4k+2$

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Abstract: In this paper, we provide the combinatorial interpretations of Rogers–Ramanujan type identities involving double sums series related to moduli $4k$ and $4k+2$ using $(n+t)$–color partitions.

Keywords and Phrases: $(n+t)$–color partitions, combinatorial interpretations, Rogers–Ramanujan type identities.

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1. Introduction

The following two celebrated identities, known as Rogers–Ramanujan Identities (RRIs), were discovered by Rogers in [9] and later rediscovered by Ramanujan:

$$\begin{aligned}\sum_{n=0}^{\infty}\frac{q^{n^2}}{(q,q)_n} &= \frac{(q^2,q^3;q^5)_\infty}{(q;q)_\infty},\\ \sum_{n=0}^{\infty}\frac{q^{n(n+1)}}{(q,q)_n} &= \frac{(q,q^4;q^5)_\infty}{(q;q)_\infty},\end{aligned}$$

where we have used the standard q–series notation

$$(x_1,x_2,\cdots,x_r;q)_\infty=\prod_{j=0}^{\infty}(1-x_1q^j)\cdots(1-x_rq^j).$$