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TWO PHASE STRUCTURE OF THE CONDENSATION BOUNDARY LAYER ON AN ACCELERATED PLATE

Shubha Devi Yadav

Department of mathematics
D.A.V.P.G. College, Lucknow, (U.P.) India
E-mail: shubha.yadav70@gmail.com

Dedicated to Prof. K. Srinivasa Rao on his 75th ***Birth Anniversary***

Abstract: In this paper, the condensation heat transfer and the structure of the dispersed, two-phase, two-component boundary layer are studied under forced convection condition on a flat plate moving with constant velocity.

Keywords and Phrases: Two phase , two component flow, boundary layer,etc.

2010 Mathematics Subject Classification: 80A22, 76T15, 35L65.

1. Introduction

Forced convection condensation on a flat plate in the presence of a non-condensing gas has been studied by many researchers. Sparrow(1967), in his work, assumed that the vapour was saturated in the main stream and on the interface. But the probability of droplet formation has not been considered. Hijikata and Mori in (1973) assumed that the vapour is saturated throughout the boundary layer and that droplets appear to avoid any metastable state of vapours. They neglected the heat resistance of the film, assuming that the heat resistance of the film, assuming that the temperature of the interface was equal to the temperature of the plate of surface. Taking all these assumptions Legay in (1984) and Leagay and Prunet in (1985, 86) solved the two-phase boundary layer equations, numerically by a finite difference method.

Poinsot and Huetz in (1985) and Sekulic in (1985) proposed another approach to the thermodynamic modeling of the boundary layer, for somewhat different problems, Poinsot and Huetz (1985) studied the condensation of quiescent vapour in the presence of non condensing gas on a cylindrical surface. Sekulrc (1985) investigated the problem of free connection condensation in the presence of non-condensing gas on a vertical cryosurface.

It would be seen that the physical assumptions adopted in Hijikata and Mori (1973), Legay and Prunet (1985-86) were the best one for the problems of force convection condensation in the presence of non-condensing gas. However, Hijikata and Mori [2] pointed out that in case of Le <1 and for low temperature differences, a condensing vapour is in a superheated state in the single phase boundary layer. However, they did not show any numerical results to support such a statement. Thereafter, no attention has been given to this problem in Legay (1983) and Legay and Prunet (1985-86), where no numerical value of droplet mass fraction were presented. At the same time, droplets were not observed in the experiments with a steam-air mixture Legay Dessesquelles (1984) unless the temperature difference was > 20K.

These results compelled researchers to deal with forced convention condensation in the presence of n on-condensing gas to saturated carefully the intrinsic consistency of the model based on Matuszkiewiez and Vernier (1990) determined the conditions which should be fulfilled in order that vapour be saturated all though the boundary layer.

In this paper, the main aim is to check the effect of the motion of the plate on the conditions which should be fulfilled in order that the vapour be saturated all through the boundary layer.

The equations describing the homogeneous, two-phase, two-component boundary layer are presented. After that, liquid film equations are written for the thin film approximation. Then, balance equations on the interface between the liquid film and dispersed, two-component medium are presented.

The whole problem has been treated in two different ways. Firstly, an approximate analytic relation between temperature and droplet mss fraction is found, Secondly the system of equations with appropriate boundary conditions is numerically solved and profiles of droplet mass fraction are given for the steam-air mixture.

In the end, the results of this paper are discussed and conclusions are formulated with special reference to the modeling of condensation boundary layers in the presence of non-condensing gas along a plate moving with a constant velocity U.

Formulation of The Problem

Consider a homogeneous fluid mixture, consisting of non-condensing gas, saturated vapours and finely dispersed droplets of the same substance flowing along a flat plate moving with a constant velocity U. The uniform temperature of the plate is kept lower than that of the main flow. So the vapours condensing on the plate and the liquid film arising on it is dragged by the shear stress. A laminar boundary arises in the region of the dispersed, two-phase,two-component medium. In the whole system only liquid film is affected by the motion of the plate.

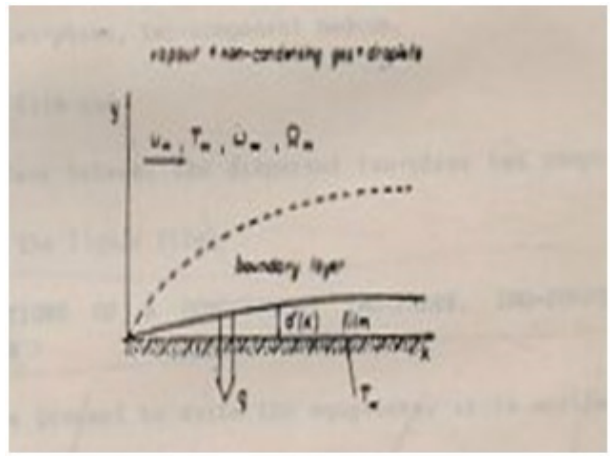

Figure 1. Physical model of coordinate system

Firstly we write the balance equations for:
1. Dispersed two-phase,two-component medium
2. The liquid film
3. The interface between the dispersed two-phase two-component medium and the liquid film.

General Equations Of A Homogeneous Two-Phase, Two-Component Boundary

Based on the eight assumptions defined as earlier, the equations for homogeneous two-phase, two-component boundary layer are as follows:

Continuity Equation:

$$\frac{dP}{dt} + \rho\left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right] = 0 \tag{1}$$

Momentum Equation:

$$\rho\frac{du}{dt} = \frac{\partial}{\partial y}\left[\mu\frac{\partial u}{\partial y}\right] \tag{2}$$

Diffusion Equation:

$$\rho(1-\Omega)\frac{dw}{dt} = \frac{\partial}{\partial y}\left[\rho(1-\Omega)D\frac{\partial w}{\partial y}\right] - \rho(1-w)\frac{d\Omega}{dt} \tag{3}$$

and

Thermal Energy Equation:

$$\rho C\frac{dT}{dt} = \rho L\frac{\partial\Omega}{\partial t}\left[\lambda\frac{\partial T}{\partial y} + \frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right)\right] \tag{4}$$

where $\frac{d}{dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial z} + v\frac{\partial}{\partial y}$, t = time, x,y = cartesian coordinates, u = velocity component parallel to the wall, v = velocity component perpendicular to the wall, μ = dynamic viscosity, D = binary diffusivity, λ thermal conductivity, c = heat capacity, L = latent heat of evaporation, T = temperature.

Mostly, the dropt mass fraction is many times less than the vapour mass fraction. So, hereafter, the source term in the diffusion equation will be neglected.

Equations of The Liquid Film

Thin film approximation, Cess (1960) Sparrow et al. (1967) are adopted, to simplify the conservation equations of the liquid film. In accordance with this approximations the inertia terms in the momentum equation and convective terms in the thermal equations are neglected. The stationary equation of the liquid film are:

Continuity:

$$\frac{\partial u_F}{\partial x} + \frac{\partial v_F}{\partial y} = 0 \tag{5}$$

Momentum:

$$\frac{\partial^2 u_F}{\partial x^2} = 0 \tag{6}$$

and

Thermal Energy

$$\frac{\partial^2 T_F}{\partial y^2} = 0 \tag{7}$$

Interface Balance Equations

Along with the equations of the homogeneous two-phase, two component boundary layer and the equation of the liquid film, the interface balance equations are given. In the boundary layer approximations and for a slightly curved film surface as in our case, the interface balance equations take the following form:

Total mass Equation:

$$\rho\left[u\frac{d\delta}{dx} - v\right] = \rho L\left[u_F\frac{d\delta}{dx} - v_F\right] \tag{8}$$

Momentum Equation:

$$\mu\frac{\partial u}{\partial y} = \mu L\frac{\partial u_F}{\partial y} \tag{9}$$

Mass-Transport of non-condensing gas Equation

$$u\frac{d\delta}{dx} - v = \frac{1}{1-w}D\frac{\partial w}{\partial y} \tag{10}$$

and

Thermal Energy Equation:

$$(1-\Omega)\rho\left[u\frac{d\delta}{dx} - v\right]L(T_i) + \lambda\frac{\partial T}{\partial y} - \lambda_L\frac{\partial T_F}{\partial y} = 0 \tag{11}$$

where, δ is the film thickness and index i refers to the interface. The interface balance equations are incomplete unless they are supported by the jump condition. Here, continuity conditions (zero jumps) are adopted.

In this problem, two continuity conditions are necessary. They are as follows:

(i) The continuity of tangential component of velocity, is given by

$$u = u_F \tag{12}$$

(ii) The condition of temperature is given by

$$T = T_F \tag{13}$$

These two continuity conditions are equivalent to the assumption that the thermodynamic process is reversible.

Relation Between Temperature and Droplet Mass Fraction

Here, the vapours are saturated throughout the boundary and for saturated vapours the vapour mass fraction and temperature are not independent quantities. The relation between vapour mass fraction and temperature is given by the following relation (equation):

$$w(T) = \frac{mP_v(T)}{\rho - (1-m)P_v(T)} \tag{14}$$

where $m = \dfrac{M_v}{M_A}$, $P_v(T)$= vapour saturation pressure, M_v= molar mass of vapour, M_A = molar mass of non-condensing gas.

If we take $\Omega << w$, then the source term in the diffusion equation may dropped. In the diffusion equation (3) and thermal equation (4) all the terms including gradient of the transport coefficients are neglected. Since, vapour mass fraction is a function of temperature, we may write:

$$\frac{\partial^2 T}{\partial y^2} = \frac{1}{w_1}\frac{\partial^2 w}{\partial y^2} - \frac{w_2}{w_1^2}\left[\frac{\partial w}{\partial y}\right]^2 \tag{15}$$

where, w_1 = is first temperature derivatives of w, w_2 = is second temperature derivative of w.

Here, the term $[w_1/w_1^3(\partial w/\partial y)^2]$ is quadratic in the gradient of vapour mass fraction, so it may be neglected in case of a small temperature difference. So, we have:

$$\frac{\partial^2 T}{\partial y^2} = \frac{1}{w_1}\frac{\partial^2 w}{\partial y^2} \tag{16}$$

Now, from diffusion equation (3) we have:

$$\frac{1}{D}\frac{dw}{dt} = \frac{\partial^2 w}{\partial y^2} \tag{17}$$

From equation (16) and (17) we have:

$$\frac{\partial^2 T}{\partial y^2} = \frac{1}{Dw_1}\frac{\partial w}{\partial t} \tag{18}$$

Now substituting equation (18) in the thermal energy equation (4), we get the following differential equation:

$$(1 - L_e)\frac{\partial T}{\partial t} = \frac{1}{c}\frac{\partial \Omega}{\partial t} \tag{19}$$

where $L_e = \dfrac{\lambda}{D\rho c}$.

Integrating of equation (19) we get:

$$\Omega - \Omega_\infty = (1 - L_e)(C/L)(T - T_\infty) \tag{20}$$

As we know, in the condensation boundary layers the temperature difference $(T - T_\infty)$ is always negative. So there are three different cases as follows:

1. For $L_e < 1$ (e.g. a steam-air mixture) the droplet mass fraction decreases as the interface is approached).

2. For $L_e > 1$ (e.g. a mixture of air and ethyl alcohol vapour, the droplet mass fraction increases as the interface is approached;

3. For $L_e = 1$, the droplet mass fraction is constant.

The detailed analysis of thermal energy equation (4) shows that the non-linear term in the vapour mass fraction gradient, which has been neglected in equation (15) in derivation of (20) is predominant for high temperature differences. Since, this non-linear term is of opposite sign to the sign of other terms in the thermal energy equation. So for $L_e < 1$, it is responsible for the raising of droplet mass fraction to positive values near the interface.

Numerical Analysis

The equations of the condensation boundary layer must be supported by appropriate boundary conditions, so as to describe the system completely. The conditions for the main flow are obtained by fixing vapour mass fraction w_∞ and the temperature T_∞. Thus, main flow conditions are:

$$u = u_\infty, T = T_\infty, w = w_\infty, \Omega = \Omega_\infty \tag{21}$$

The boundary conditions at the wall are:

$$u_F = U, v_F = 0, T_F = T \tag{22}$$

Considering the stationary procedure, and excluding the region near the loading edge, a self similar transformation is applied to the system of condensation boundary layer equations. It reduces the system to a system of ordinary differential equations.

Transformations For The Dispersed Two-Phase, Two Component Boundary Layer

The following transformations are applied to the dispersed two-phase, two-component boundary layer equations:

$$\xi = (y - \delta)\left[\frac{u_\infty}{v_x}\right] \tag{23}$$

$$\psi = (u_\infty - v_x)^{1/2} F(\xi)$$

where v is kinetic viscosity.

Applying transformation (23) to equation (1)-(4) we get the following ordinary differential equations:

$$F''' + \frac{1}{2} F F'' = 0 \tag{24}$$

$$\Phi'' + \frac{1}{2} Sc F \Phi' = 0 \tag{25}$$

and

$$\frac{L}{c}\Omega' = (1 - L_e)\frac{w_\infty - w_i}{w_1}\Phi' - \left[\frac{2w_2(w_\infty - w_i)^2}{Prw_1^2}\right]\frac{\Phi'^2}{F} \tag{26}$$

where Sc is the Schmidt number, Pr is the Prandtt number, denotes the derivative and $\Phi = \dfrac{w - w_i}{w_\infty - w_i}$

Transformation For The Liquid Film Equations

Following transformations are applied to the liquid film equations (5)-(7), along a flat plate moving with uniform velocity U.

$$\eta = y\left[\frac{u_\infty}{v_L x}\right]^{1/2}, \quad \psi_F = (u_\infty v_L x)^{1/2} f(\eta) \tag{27}$$

Applying boundary (27) to equations (5) we get the following ordinary differential equations:

$$f(\eta) = \eta\frac{2U}{u_\infty} \tag{28}$$

and

$$\frac{T - T_w}{T_i - T_w} = \frac{\eta}{\eta_\delta} \tag{29}$$

where $\eta\delta$ is the value of η for $y = \delta$.

The interface balance equations (8)-(11) are transformed with the aid of (23) and (27), Transformation of equation (8) with the aid of (28) gives the following equation

$$\delta\frac{d\delta}{dx} - \frac{1}{4}\frac{\delta^2}{x} = \frac{1}{2}\frac{\rho(vv_L)^{1/2}F(0)}{\rho_l u_\infty f''_w(0)} \tag{30}$$

Solving equation (30) for δ

$$\delta(x) = \left[\frac{2\rho(vv_L)^{1/2}F(0)}{\rho_L u_\infty f''_\infty(0)}x\right]^{1/2} \tag{31}$$

Transformation of interface momentum balance equation gives the following equation

$$Rf''_w(0) = F''(0) \tag{32}$$

where $R = \dfrac{[\rho_L\mu_L]}{\rho\mu}$.

Transformation of the interface balance equations of mass of non-condensing gas (10) gives the following equation:

$$\frac{1}{2}ScF(0) = \frac{w_\infty - w_i}{1 - w_i}\Phi'(0) \tag{33}$$

Similarly transformation of the interracial balance equation of thermal energy (11), gives the following equation:

$$\frac{1}{2}F(0)\left[(1-\Omega_i) + \frac{\lambda(1-w_i)}{\rho DL(T_i)w_i()T_i}\right] = \frac{\lambda_L(T_i - T_w)}{\mu_L L(T_i)}\sqrt{R}\frac{1}{\eta_\delta} \tag{34}$$

where

$$\eta_\delta = \left[\frac{2F(0)}{F''(0)}\right]^{1/2} \tag{35}$$

Transformation of the continuity condition leads to

$$F'(0) = \left[\frac{2F(0)F''(0)}{R}\right]^{1/2} \tag{36}$$

Also, the main flow conditions are:

$$F'(\infty) = 1, \Phi(0) = 0, \Phi(\infty) = 1, \Omega\infty = \Omega_\infty \tag{37}$$

Now, the whole problem is reduced to solving the equations of the dispersed two-phase two-component boundary layer [(24)-(26)] with the aid of relation (14) with non-linear boundary conditions [(33)-(36)].

This sixth order system of ordinary differential equations [(24)-(26)] was solved b y fourth order Runge-Kutta method (Matuszkiwiez) et al (1989).

Results and Discussion

For a steam-air mixture (i.e. $L_e < 1$) calculations were made with the main-stream temperature $T_\infty = 37312$ for three different droplet of vapour mass fraction i.e. 0.3, 0.6 and 0.9) and droplet mass fraction taken to be constant at 0. The range of temperature difference $(T_\infty - T_w)$ was taken between 5k and 30k. Using the above values, the observations made are presented in the following graphs:

About Graph II

This graph shows the variation of non-dimensional temperature difference between the interface and the wall, with the change in the overall temperature difference $(T - T_w)$, for three different values of the main stream vapour mass fraction.

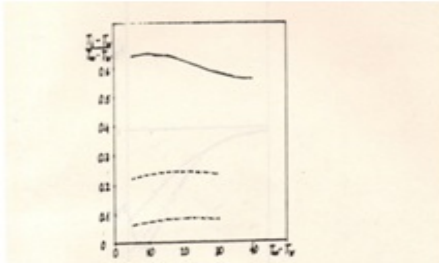

Figure 2 Variation of $(T_i - T_w)$ with $(T_\infty - T_w)$ for a steam-air system: $T_\infty = 373K$, $\Omega = 0$: $w_\infty = 0.9$ (———), $w_\infty = 0.6$ (------), $w_\infty = 0.3$ (—·—·—·—).

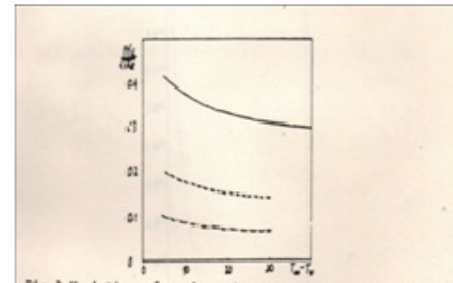

Fig.3 Variation of condensation heat transfer with temperature difference for a steam-air system: $T_\infty = 373K$, $\Omega_\infty = 0$; $w_\infty = 0.9$ (———), $w_\infty = 0.6$ (-------), $w_\infty = 0.3$ (- . . . -)

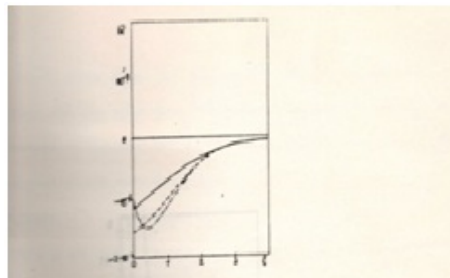

Figure 4. Droplet mass fraction profiles within the condensation boundary layer for a steam-air system (Le < 1): $T_\infty = 373K$, $w_\infty = 0.6$, $\Delta T = 10K$ (———), $\Delta T = 20K$ (-----), $\Delta T = 30K$ (—·—·—·—)

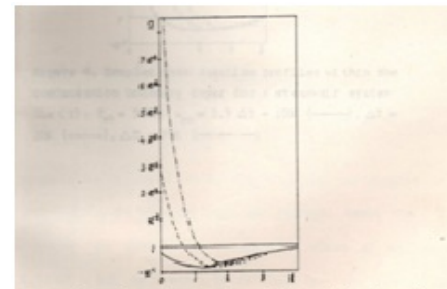

Figure 5. Droplet mass fraction profiles within the condensation boundary layer for a steam-air system (Le < 1): $T_\infty = 373K$, $w_\infty = 0.6$; $\Delta T = 10K$ (———), $\Delta T = 20K$ (-----), $\Delta T = 30K$, (—·—·—·—)

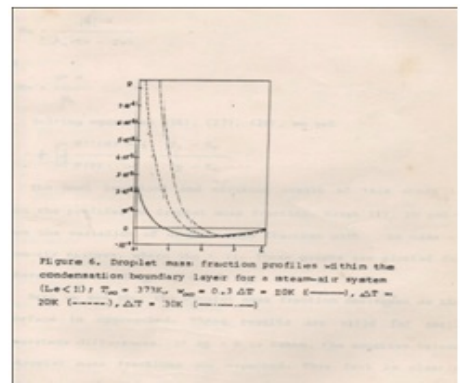

Figure 6. Droplet mass fraction profiles within the condensation boundary layer for a steam-air system (Le < 1): $T_\infty = 373K$, $w_\infty = 0.3$ $\Delta T = 10K$ (———), $\Delta T = 20K$ (-----), $\Delta T = 30K$ (—·—·—·—)

This graph shows that the temperature difference between the interface and the wall does not depend strongly on the overall temperature difference but it does depend strongly on the main stream vapour mass fraction.

About Graph II, This graph shows the heat flux transferred to the wall. The results of calculations are expressed in terms of the quantity $Nu/(Re)^{1/2}$ vs the temperature difference $(T_\infty - T_w)$, where $Nu = \dfrac{|q|x}{\lambda_L(T_\infty - T_w)}$; q is heat flux and $Re = \dfrac{u_\infty x}{v_L}$.

Solving equations (24), (25) and (26), we get:

$$\frac{Nu}{Re} = + \left[\frac{1}{2}\frac{F''(0)}{F(0)}\right]^{1/2} \frac{T_i - T_w}{T_\infty - T_w} \tag{38}$$

Graph III, IV and V shows the variation of droplet mass fraction with, in case of steam-air mixture, where $(Le < 1)$. These graphs are plotted for different values of w_∞; (i.e. $w_\infty = 0.3, w_\infty = 0.6, w_\infty = 0.9$). It is clear that the deoplet mass fraction decreases as the interface is approached. These results are valid for small temperature differences . For
$\Omega_\infty = 0$, the negative values of droplet mass fraction are expected (as seen in graph III, IV and V).
Although negative values of droplet mass fraction are physically meaningless, it may be concluded that the vapour is superheated in this region. Hence, the saturation condition is not to be imposed throughout the boundary layer for $Le < 1$, when there are not droplets in the main flow. Thus for $Le > 1$, the conditions $\Omega \geq 0$ should be satisfied all along the boundary layer, while for $Le < 1$, it is satisfied only for the following condition.

$$\Omega_\infty > \Omega_\infty^{min}$$

where

$$\Omega_\infty^{min} = (1 - L_e)\frac{C}{L}\left[\frac{T_\infty - T_i}{L(T_\infty)}\right]$$

Also, we see from the model equation derived that there is a considerable change in the film thickness.

Conclusions

From the results we can say that the main conclusions of this paper are:

1. As the temperature decreases in the condensation boundary layer, the droplet mass fraction decreases slightly in an external part of the boundary and droplet mass fraction increase rapidly neat the interface for $le < 1$.

2. The heat flux transferred to the wall is only slightly influenced by droplets presents in the condensation boundary layer.

3. The assumption that $T_i = T_w$ is not acceptable unless the vapour mass fraction is very low, e.g. < 0.1 for the steam air mixture.

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