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A NOTE ON FRACTIONAL DERIVATIVE AND ITS APPLICATIONS

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Dedicated to Prof. K. Srinivasa Rao on his 75^{th} Birth Anniversary

Abstract: In this paper, starting from the historical developments of fractional calculus, certain results regarding fractional calculus have been discussed. These results have been further used to establish transformation formulae for ordinary hypergeometric series as well as for q-hypergeometric series.

Keywords and Phrases: Fractional derivative, fractional q-derivative, transformation formula, hypergeometric series, q-hypergeometric series.

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1. Introduction, Notations and Definitions

The generalized hypergeometric function ${}_pF_q(x)$ is defined as

$$ {}_pF_q\left[\begin{array}{c} a_1, a_2, ..., a_p; x \\ b_1, b_2, ..., b_q \end{array}\right] = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k ... (a_p)_k x^k}{(b_1)_k (b_2)_k ... (b_q)_k k!}. \tag{1.1} $$

When $q = p$, this series converges for $|x| < \infty$, but when $p = q + 1$, convergence occurs for $|x| < 1$. In (1.1) the Pochhammer symbol $(a)_k$ is defined by $(a)_0 = 1$ and for $k \geq 1$ by $(a)_k = a(a+1)...(a+k-1)$. However, for all integers k we write simply $(a)_k = \dfrac{\Gamma(a+k)}{\Gamma(a)}$.

We shall also use the notation,