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CERTAIN CLASS OF EULERIAN INTEGRALS WITH THE MULTIVARIABLE I-FUNCTION DEFINED BY NAMBISAN

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Dedicated to Prof. K. Srinivasa Rao on his 75^{th} ***Birth Anniversary***

Abstract: In this paper, first we evaluate a class of MacRobert's integral associated with the multivariable I-function defined by Nambisan et al [3], secondly we evaluate a class of MacRobert's with. the generalized incomplete hypergeometric function, a general class of polynomials and the multivariable I-function defined by Nambisan et al [3]. We will study several particular cases.

Keywords and Phrases: General class of polynomials, generalized incomplete hypergeometric function, multivariable I-function, Srivastava-Daoust function, multivariable H-function.

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1. Introduction and preliminaries

In this document, we derive an integral involving the generalized incomplete hypergeometric function, a class of multivariable polynomials and the multivariable I-function. For this multivariable I-function, we adopt the contracted notations.

The multivariable I-function defined by Nambisan et al [3] is an extension of the multivariable H-function defined by Srivastava et al [7].

The multivariable I-function is defined in term of multiple Mellin-Barnes type integral :

$$I(z_1,...,z_r) = I^{0,n:m_1,n_1;...;m_r,n_r}_{p,q:p_1,q_1;...;p_r,q_r}\left(\begin{array}{c|c} z_1 & (a_j;\alpha_j^{(1)},...,\alpha_j^{(r)};A_j)_{1;p} : \\ \cdot & \\ \cdot & \\ \cdot & \\ z_r & (b_j;\beta_j^{(1)},...,\beta_j^{(r)};B_j)_{1,q} : \end{array}\right.$$

$$\left.\begin{matrix}(c_j,\gamma_j^{(1)};C_j^{(1)})_{1,p_1};...;(c_j^{(r)},\gamma_j^{(r)};C_j^{(r)})_{1;p_r}\\ \\ \\ (d_j^{(1)},\delta_j^{(1)};D_j^{(1)})_{1,q_1};...;(d_j^{(r)},\delta_j^{(r)};D_j^{(r)})_{1,q_r}\end{matrix}\right) \tag{1.1}$$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,...,s_r)\prod_{i=1}^{r}\theta_i(s_i)z_i^{s_i}ds_1...ds_r, \tag{1.2}$$

where $\phi(s_1,...,s_r)$, $\theta_i(s_i)$, $i=1,...,r$ are given by;

$$\phi(s_1,...,s_r)=\frac{\prod_{j=1}^{n}\Gamma^{A_j}\left(1-a_j+\sum_{i=1}^{r}\alpha_j^{(i)s_j}\right)}{\prod_{j=n+1}^{p}\Gamma^{A_j}\left(a_j-\sum_{i=1}^{r}\alpha_j^{(i)}s_j\right)\prod_{j=1}^{q}\Gamma^{B_j}\left(1-b_j+\sum_{i=1}^{r}\beta_j^{(i)}s_j\right)} \tag{1.3}$$

$$\theta_i(s_i)=\frac{\prod_{j=1}^{n_i}\Gamma^{C_j^{(i)}}\left(1-c_j^{(i)}+\gamma_j^{(i)}s_i\right)\prod_{j=1}^{m_i}\Gamma^{D_j^{(i)}}\left(d_j^{(i)}-\delta_j^{(i)}s_i\right)}{\prod_{j=n_i+1}^{p_i}\Gamma^{C_j^{(i)}}\left(c_j^{(i)}-\gamma_j^{(i)}s_i\right)\prod_{j=m_i+1}^{q_i}\Gamma^{D_j^{(i)}}(1-d_j^{(i)}-\delta_j^{(i)})s_i} \tag{1.4}$$

For more details, see Nambisan et al [6].
Following the result of Braaksma [1] the I-function of r variables is analytic if :

$$U_i=\sum_{j=1}^{p}A_j\alpha_j^{(i)}-\sum_{j=1}^{q}B_j\beta_j^{(i)}+\sum_{j=1}^{p_i}C_j^{(i)}\gamma_j^{(i)}-\sum_{j=1}^{q_i}D_j^{(i)}\delta_j^{(i)}\leq 0,\quad i=1,...,r \tag{1.5}$$

The complex numbers z_i are not zero. Throughout this document, we assume the existence and absolute convergence conditions of the multivariable I-function.
The integral (2.1) converges absolutely if

$$|\arg(z_k)|<\frac{1}{2}\Delta_k\pi,\qquad k=1,...,r$$

where

$$\Delta_k=-\sum_{j=n+1}^{p}A_j\alpha_j^{(k)}-\sum_{j=1}^{q}B_j\beta_j^{(k)}+\sum_{j=1}^{m_k}D_j^{(k)}\delta_j^{(k)}-\sum_{j=m_k+1}^{q_k}D_j^{(k)}\delta_j^{(k)}$$

$$+\sum_{j=1}^{n_k} C_j^{(k)}\gamma_j^{(k)} - \sum_{j=n_k+1}^{p_k} C_j^{(k)}\gamma_j^{(k)} > 0 \tag{1.6}$$

We will use these notations for this paper,

$$X = m_1, n_1; ...; m_r, n_r; V = p_1, q_1; ...; p_r, q_r \tag{1.7}$$

$$A = (a_j; \alpha_j^{(1)}, ..., \alpha_j^{(r)}; A_j)_{1,p} \tag{1.8}$$

$$B = (b_j; \beta_j^{(1)}, ..., \beta_j^{(r)}; B_j)_{1,q} \tag{1.9}$$

$$C = (c_j^{(1)}, \gamma_j^{(1)}; C_j^{(1)})_{1,p_1}; ...; (c_j^{(r)}, \gamma_j^{(r)}; C_j^{(r)})_{1,p_r} \tag{1.10}$$

$$D = (d_j^{(1)}, \delta_j^{(1)}; D_j^{(1)})_{1,q_1}; ...; (d_j^{(r)}, \delta_j^{(r)}; D_j^{(r)})_{1,q_r} \tag{1.11}$$

the contracted form is

$$I_{p,q;V}^{0,n;X}\left(\begin{array}{c|c} z_1 & A:C \\ \cdot & \\ \cdot & ... \\ \cdot & \\ z_r & B:D \end{array}\right) \tag{1.12}$$

Srivastava and Garg [6] introduced and defined a general class of multivariable polynomials as follows

$$S_L^{h_1,...,h_u}[z_1, ..., z_u] = \sum_{R_1,...,R_u=0}^{h_1R_1+...+h_uR_u\leq L} (-L)_{h_1R_1+...+h_uR_u} B(E; R_1, ..., R_u)\frac{z_1^{R_1}...z_u^{R_u}}{R_1!...R_u!} \tag{1.13}$$

The coefficients $B(E; R_1, ..., R_u)$ are arbitrary constants, real or complex. We will note;

$$B_u = \frac{(-L)_{h_1R_1+...+h_uR_u} B(E; R_1, ..., R_u)}{R_1!...R_u!} \tag{1.14}$$

2. Generalized incomplete hypergeometric function

The generalized incomplete hypergeometric function introduced by Srivastava et al [4 page 675, Eq.(4.1)] is represented in the following manner.

$${}_p\gamma_q\left[\begin{array}{l} (e_1;\sigma), (e_2), ..., (e_p) \\ (f_1), ..., (f_q) \end{array}; z\right] = \sum_{n=0}^{\infty} \frac{(e_1;\sigma)_n (e_2)_n ... (e_p)_n z^n}{(f_1)_n ... (f_q)_n n!} \tag{2.1}$$

$|z| < 1$ where the incomplete Pochhammer symbols are defined as follows :

$$(a;\sigma)_n = \frac{\gamma(a+n;\sigma)}{\Gamma(a)} \quad (a, n \in \mathbb{C}; x \geq 0) \tag{2.2}$$

$$\gamma(s,x)=\int_0^x t^{s-1}e^{-t}dt \quad (Re(s)>0,\ x\geq 0) \tag{2.3}$$

3. Required integral

We have the following integral, see Mac Robert's [2]

Lemma

$$\int_a^b (t-a)^{\lambda-1}(b-t)^{\mu-1}[b-a+c(t-a)+d(b-t)]^{-\lambda-\mu}$$

$$\times\ {}_2F_1\left[\alpha,\beta;\mu;\frac{(b-t)(1+d)}{b-a+c(t-a)+d(b-t)}\right]dt$$

$$=\frac{(1+c)^{-\lambda}(1+d)^{-\mu}\Gamma(\lambda)\Gamma(\mu)\Gamma(\lambda+\mu-\alpha-\beta)}{(b-a)\Gamma(\lambda+\mu-\alpha)\Gamma(\lambda+\mu-\beta)} \tag{3.1}$$

valid for $Re(\mu)>0$, $Re(\lambda)>0$, $Re(\mu-\alpha-\beta)>0$, $b\neq a$, $t\in[a,b]$ and $b-a+c(t-a)+d(b-t)\neq 0$

4. Eulerian integral involving the multivariable I-function

Let $G=\Gamma(\mu)(b-a)^{-1}(1+c)^{-\lambda}(1+d)^{-\mu}$, $X=\dfrac{(t-a)(1+c)}{b-a+c(t-a)+d(b-t)}$ and $A_{n'}=\dfrac{(e_1;\sigma)_n(e_2)_n...(e_p)_n}{(f_1)_n...(f_q)_n}$

The main result to be established here is

Theorem 1

$$\int_a^b (t-a)^{\lambda-1}(b-t)^{\mu-1}[b-a+c(t-a)+d(b-t)]^{-\lambda-\mu}$$

$$\times\ {}_2F_1\left[\alpha,\beta;\mu;\frac{(b-t)(1+d)}{b-a+c(t-a)+d(b-t)}\right]I(z_1X^{\upsilon_1},...,z_rX^{\upsilon_r})dt$$

$$=I^{0,n+2;X}_{p+2,q+2;V}\left(\begin{array}{c|l} z_1 & (1-\lambda;\upsilon_1,...,\upsilon_r;1), \\ \cdots & \\ \cdots & \\ z_r & (1-\lambda-\mu+\alpha;\upsilon_1,...,\upsilon_r;1), \end{array}\right.$$

$$\left.\begin{array}{l} (1-\lambda-\mu+\alpha+\beta;\upsilon_1,...,\upsilon_r;1),A:C \\ \\ (1-\lambda-\mu+\beta;\upsilon_1,...,\upsilon_r;1),B:D \end{array}\right) \tag{4.1}$$

provided that $Re(\mu) > 0$, $Re(\lambda) > 0$, $Re(\mu - \alpha - \beta) > 0$, $b \neq a$, $t \in [a, b]$; $\upsilon_i > 0$, $i = 1, ..., r$ $b - a + c(t - a) + d(b - t) \neq 0$, the conditions (f) are verified and the conditions of existence of the multivariable function are satisfied.

Proof.

Let

$$M\{\} = \frac{1}{(2\pi\omega)^r} \int_{L_1} \cdots \int_{L_r} \psi(s_1, ..., s_r) \prod_{k=1}^{r} \theta_k(s_k)\{\}$$

We first replace the multivariable I-function by its Mellin-Barnes contour integral with the help of (1.1), we get

$$\int_a^b (t-a)^{\lambda-1}(b-t)^{\mu-1}[b-a+c(t-a)+d(b-t)]^{-\lambda-\mu}$$

$$\times\ {}_2F_1\left[\alpha, \beta; \mu; \frac{(b-t)(1+d)}{b-a+c(t-a)+d(b-t)}\right] dt$$

$$\times \left\{ \left[\frac{(t-a)(1+c)}{b-a+c(t-a)+d(b-t)}\right]^{\sum_{j=1}^{r} \upsilon_j s_j} z_1^{s_1} ... z_r^{s_r} ds_1 ... ds_r \right\} dt \tag{4.2}$$

Assuming the inversion of order of integrations in (4.2) to be permissible by absolute (and uniform) convergence of integrals involved above, we have

$$M\{(1+c)^{\sum_{j=1}^{r} s_j \upsilon_j} z_1^{s_1} ... z_r^{s_r} \int_a^b (t-a)^{\lambda + \sum_{j=1}^{r} \upsilon_j s_j - 1} (b-t)^{\mu-1} \left[b-a+c(t-a)\right.$$

$$\left. +(b-t)\right]^{-\lambda-\mu-\sum_{j=1}^{r} \upsilon_j s_j}\ {}_2F_1\left[\alpha, \beta; \mu; \frac{(b-t)(1+d)}{b-a+c(t-a)+d(b-t)}\right] dt\} ds_1 ... ds_r \tag{4.3}$$

Now evaluating the inner integral with the help of Lemma (3.1) valid for $Re(\mu) > 0$, $Re(\lambda) > 0$, $Re(\mu - \alpha - \beta) > 0$, $b \neq a$, $t \in [a, b]$ and finally interpreting the resulting Mellin-Barnes contour integrals as a multivariable I-function, we obtain the desired result.

5. Particular cases

The integral formula (4.2) has manifold generality. By specializing the various parameters and variables involved, the formula can suitable applied to derive the corresponding results involving remarkably wide range of useful functions.

(1) Putting $\beta = \mu$ in (4.1), provided that $Re(\lambda) > 0$, $Re(\mu) > 0$ and $b - a + c(t -$

$a) + d(b-t) \neq 0$; $b \neq a$; $t \in [a,b]$ $\left|\frac{(b-t)(1+d)}{b-a+c(t-a)+d(b-t)}\right| < 1$ and using the transformation

$$ {}_2F_1(a,b;b;z) = (1-z)^{-a} \tag{5.1}$$

we get

Corollary 1

$$\int_a^b (t-a)^{\lambda-\alpha-1}(b-t)^{\mu-1}[b-a+c(t-a)+d(b-t)]^{-\lambda+\alpha-\mu} I(z_1 X^{\upsilon_1},...,z_r X^{\upsilon_r})dt$$

$$= G_1 \ I^{0,n+1;X}_{p+1,q+1;V}\left(\begin{array}{c|c} z_1 & (1-\lambda+\alpha;\upsilon_1,...,\upsilon_r;1), A:C \\ \cdots & \\ \cdots & \\ z_r & (1-\lambda-\mu+\alpha;\upsilon_1,...,\upsilon_r;1), B:D \end{array}\right), \tag{5.2}$$

where $G_1 = \dfrac{\Gamma(\mu)(1+c)^{-\lambda+\alpha}(1+d)^{-\mu}}{b-a}$

(2) Putting $c = d = 0$ and $\upsilon_1 = ... = \upsilon_r = 1$, we get

Corollary 2

$$\int_a^b (t-a)^{\lambda-1}(b-t)^{\mu-1} \ {}_2F_1\left[\alpha,\beta;\mu;\frac{b-t}{b-a}\right] I(z_1 Y^{\upsilon_1},...,z_r Y^{\upsilon_r})dt = \Gamma(\mu)(b-a)^{\lambda+\mu-1}$$

$$= I^{0,n+2;X}_{p+2,q+2;V}\left(\begin{array}{c|c} z_1 & (1-\lambda;1,...,1;1),(1-\lambda-\mu+\alpha+\beta;1,...,1;1), A:C \\ \cdots & \\ \cdots & \\ z_r & (1-\lambda-\mu+\alpha;1,...,1;1),(1-\lambda-\mu+\beta;1,...,1;1), B:D \end{array}\right) \tag{5.3}$$

where $Y = \dfrac{b-t}{b-a}$

(3) Putting $\lambda = \mu = \dfrac{1}{2}$; $c = d = 1$; $\upsilon_1 = ... = \upsilon_r = 1$ and $\alpha = 0$, we get

Corollary 3

$$\int_a^b [(t-a)(b-t)]^{-\frac{1}{2}} \ {}_2F_1\left[\alpha,\beta;\mu;\frac{b-t}{b-a}\right] I(z_1 Y^{\upsilon_1},...,z_r Y^{\upsilon_r})dt = \sqrt{\pi}$$

$$I^{0,n+1;X}_{p+1,q+1;V}\left(\begin{array}{c|c} z_1 & (\frac{1}{2};1,...,1;1), A:C \\ \cdots & \\ \cdots & \\ z_r & (0;1,...,1;1), B:D \end{array}\right) \tag{5.4}$$

6. Multivariable I-function with the extension of Hurwitz generalized hypergeometric function and a class of polynomials

In this section, we evaluate a class of MacRobert's integral associated with the Generalized incomplete hypergeometric function, a general class of polynomials and the multivariable I-function defined by Nambisan et al [2], we have

Theorem 2

$$\int_a^b (t-a)^{\lambda-1}(b-t)^{\mu-1}[b-a+c(t-a)+d(b-t)]^{-\lambda-\mu}$$

$$\times\ {}_2F_1\left[\alpha,\beta;\mu;\frac{(b-t)(1+d)}{b-a+c(t-a)+d(b-t)}\right]\ {}_p\gamma_q\left[\begin{matrix}(e_1;\sigma),(e_2),...,(e_p)\\(f_1),...,(f_q)\end{matrix}\ zX^{\xi}\right]$$

$$S_L^{h_1,...,h_u}(y_1X^{\gamma_1},...,y_uX^{\gamma_u})I(z_1X^{\upsilon_1},...z_rX^{\upsilon_r})dt = G\sum_{n'=0}^{\infty}\sum_{R_1,...,R_u=0}^{h_1R_1+...+h_uR_u\leq L} B_u y_1^{R_1}...y_u^{R_u}$$

$$\frac{A_{n'}z^{n'}}{n'!}\ I_{p+2,q+2;V}^{0,n+2;X}\left(\begin{matrix}z_1\\ \cdots\\ \cdots\\ z_r\end{matrix}\ \middle|\ \begin{matrix}A,(1-\lambda-n'\xi-\sum_{i=1}^u R_i\gamma_i;\upsilon_1,...,\upsilon_r;1),\\ ...\\ B,(1-\lambda-\mu+\alpha-n'\xi-\sum_{i=1}^u R_i\gamma_i;\upsilon_1,...,\upsilon_r;1),\end{matrix}\right.$$

$$\left.\begin{matrix}(1-\lambda-\mu+\alpha+\beta-n'\xi-\sum_{i=1}^u R_i\gamma_i;\upsilon_1,...,\upsilon_r;1):C\\ \\ (1-\lambda-\mu+\beta-n'\xi-\sum_{i=1}^u R_i\gamma_i;\upsilon_1,...,\upsilon_r;1):D\end{matrix}\right) \tag{6.1}$$

provided that $Re(\mu)>0$, $Re(\lambda)>0$, $Re(\mu-\alpha-\beta)>0$, $b\neq a$, $t\in[a,b]$; $\upsilon_i>0$, $i=1,...,r$; $\gamma_j>0$, $j=1,...,u$, $\xi>0$, $b-a+c(t-a)+d(b-t)\neq 0$, the series is absolutely and uniformly convergent and the conditions of the multivariable function are satisfied.

Proof.

Let

$$M\{\} = \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,...,s_r)\prod_{k=1}^r\theta_k(s_k)\{\}$$

To prove (6.1), expressing the general class of polynomials of several variables $S_L^{h_1,...,h_u}[.]$ in series with the help of (1.11), The generalized incomplete hypergeometric function ${}_p\gamma_q$ in series with the help of (2.1) and the I-function of r variables in Mellin-Barnes contour integral with the help of (1.1), changing the order of integration and summation (which is easily seen to be justified due to the absolute convergence of the integral and the summations involved in the process), we get

$$\sum_{n'=0}^{\infty}\sum_{R_1,...,R_u=0}^{h_1R_1+...+h_uR_u\leq L} B_u y_1^{R_1}...y_u^{R_u}\frac{A_{n'}z^{n'}}{n'!}\int_a^b (t-a)^{\lambda-1}(b-t)^{\mu-1}$$

$$[b-a+c(t-a)+d(b-t)]^{-\lambda-\mu}\ {}_2F_1\left[\alpha,\beta;\mu;\frac{(b-t)(1+d)}{b-a+c(t-a)+d(b-t)}\right]$$

$$\times\left\{\left[\frac{(t-a)(1+c)}{b-a+c(t-a)+d(b-t)}\right]^{n'\xi+\sum_{j=1}^{u}R_j\gamma_j+\sum_{j=1}^{r}v_js_j} z_1^{s_1}...z_r^{s_r}ds_1...ds_r\right\}dt \quad (6.2)$$

Assuming the inversion of order of integrations in (6.2) to be permissible by absolute (and uniform) convergence of integrals involved above, we have

$$\sum_{n'=0}^{\infty}\sum_{R_1,...,R_u=0}^{h_1R_1+...+h_uR_u\leq L} B_u y_1^{R_1}...y_u^{R_u}\frac{A_{n'}z^{n'}}{n'!}M\left\{(1+c)^{n'\xi+\sum_{j=1}^{u}R_j\gamma_j+\sum_{j=1}^{r}s_jv_j}\ z_1^{s_1}...z_r^{s_r}\right.$$

$$\int_a^b (t-a)^{\lambda+n'\xi+\sum_{j=1}^{u}R_j\gamma_j+\sum_{j=1}^{r}v_js_j-1(b-t)^{\mu-1}}\ {}_2F_1\left[\alpha,\beta;\mu;\frac{(b-t)(1+d)}{b-a+c(t-a)+d(b-t)}\right]$$

$$[b-a+c(t-a)+d(b-t)]^{-\lambda-\mu-n'\xi-\sum_{j=1}^{u}R_j\gamma_j-\sum_{j=1}^{r}u_js_j}\ dt\}ds_1...ds_r \quad (6.3)$$

To evaluate the inner integral, use the Lemma valid for $Re(\mu)>0$, $Re(\lambda)>0$, $Re(\mu-\alpha-\beta)>0$, $b\neq a$, $t\in[a,b]$, $b-a+c(t-a)+d(b-t)\neq 0$, the multiple series is absolutely and uniformly convergent and finally interpreting the resulting Mellin-Barnes contour integrals as a multivariable I-function, we obtain the desired result.

7. Multivariable H-function

If $A'_j = B'_j = C_j^{'(i)} = D_j^{'(i)} = 1$, the multivariable I-function defined by Nambisan et al [3] reduces to multivariable H-function defined by Srivastava et al [6] and we obtain the following multiple integral transform

Corollary 4

$$\int_a^b (t-a)^{\lambda-1}(b-t)^{\mu-1}[b-a+c(t-a)+d(b-t)]^{-\lambda-\mu}$$

$$\times\ {}_2F_1\left[\alpha,\beta;\mu;\frac{(b-t)(1+d)}{b-a+c(t-a)+d(b-t)}\right]\ {}_p\gamma_q\left[\begin{matrix}(e_1;\sigma),(e_2),...,(e_p)\\(f_1),...,(f_q)\end{matrix}\ zX^{\xi}\right]$$

$$S_L^{h_1,...,h_u}(y_1X^{\gamma_1},...,y_uX^{\gamma_u})H(z_1X^{\upsilon_1},...z_rX^{\upsilon_r})dt = G\sum_{n'=0}^{\infty}\sum_{R_1,...,R_u=0}^{h_1R_1+...+h_uR_u\leq L} B_u y_1^{R_1}...y_u^{R_u}$$

$$\frac{A_{n'}z^{n'}}{n'!}\ H_{p+2,q+2;V}^{0,n+2;X}\left(\begin{array}{c|l} z_1 & A,(1-\lambda-n'\xi-\sum_{i=1}^{u}R_i\gamma_i;\upsilon_1,...,\upsilon_r), \\ \cdots & \cdots \\ \cdots & B,(1-\lambda-\mu+\alpha-n'\xi-\sum_{i=1}^{u}R_i\gamma_i;\upsilon_1,...,\upsilon_r), \\ z_r & \end{array}\right.$$

$$\left.\begin{array}{c} (1-\lambda-\mu+\alpha+\beta-n'\xi-\sum_{i=1}^{u}R_i\gamma_i;\upsilon_1,...,\upsilon_r):C \\ \\ (1-\lambda-\mu+\beta-n'\xi-\sum_{i=1}^{u}R_i\gamma_i;\upsilon_1,...,\upsilon_r):D \end{array}\right) \tag{7.1}$$

under the same conditions and notations that (6.1) with $A'_j = B'_j = C'^{(i)}_j = D'^{(i)}_j = 1$.

8. Srivastava-Daoust function

If

$$B(L;R_1,...,R_u) = \frac{\prod_{j=1}^{\bar{A}}(a_j)_{R_1\theta'_j+...+R_u\theta_j^{(u)}}\prod_{j=1}^{B'}(b'_j)_{R_1\phi'_j}\cdots\prod_{j=1}^{B^{(u)}}(b_j^{(u)})_{R_u\phi_j^{(u)}}}{\prod_{j=1}^{\bar{C}}(c_j)_{m_1\psi'_j+...+m_u\psi_j^{(u)}}\prod_{j=1}^{D'}(d'_j)_{R_1\delta'_j}\cdots\prod_{j=1}^{D^{(u)}}(d_j^{(u)})_{R_u\delta_j^{(u)}}} \tag{8.1}$$

then the general class of multivariable polynomial $S_L^{h_1,...,h_u}[z_1,...,z_u]$ reduces to generalized Lauricella function defined by Srivastava et al [5].

$$F_{\bar{C}:D';...;D^{(u)}}^{1+\bar{A}:B';...;B^{(u)}}\left(\begin{array}{c|c} z_1 & [(-L);R_1,...,R_u][(a);\theta',...,\theta^{(u)}]:[(b');\phi'];...;[(b^{(u)});\phi^{(u)}] \\ \cdots & \\ \cdots & \\ z_u & [(c);\psi',...,\psi^{(u)}]:[(d');\delta'];...;[(d^{(u)});\delta^{(u)}] \end{array}\right) \tag{8.2}$$

and we have the following formula

Corollary 5

$$\int_a^b (t-a)^{\lambda-1}(b-t)^{\mu-1}[b-a+c(t-a)+d(b-t)]^{-\lambda-\mu}$$

$$\times\ {}_2F_1\left[\alpha,\beta;\mu;\frac{(b-t)(1+d)}{b-a+c(t-a)+d(b-t)}\right]\ {}_p\gamma_q\left[\begin{array}{c}(e_1;\sigma),(e_2),...,(e_p)\\(f_1),...,(f_q)\end{array}\ zX^{\xi}\right]$$

$$F_{\bar{C}:D';...;D^{(u)}}^{1+\bar{A}:B';...;B^{(u)}}\left(\begin{array}{c|c} y_1X^{\gamma_1} & [(-L);R_1,...,R_u][(a);\theta',...,\theta^{(u)}]:[(b');\phi'];...;[(b^{(u)});\phi^{(u)}] \\ \cdots & \\ \cdots & \\ y_uX^{\gamma_u} & [(c);\psi',...,\psi^{(u)}]:[(d');\delta'];...;[(d^{(u)});\delta^{(u)}] \end{array}\right)$$

$$I(z_1X^{\upsilon_1},...,z_rX^{\upsilon_r})dt = G\sum_{n'=0}^{\infty}\sum_{R_1,...,R_u=0}^{h_1R_1+...+h_UR_u\leq L}\frac{A_{n'}z^{n'}}{n'!}y_1^{R_1}...y_u^{R_u}B'_u$$

$$I_{p+2,q+2;V}^{0,n+2;X}\left(\begin{array}{c|l} z_1 & A,(1-\lambda-n'\xi-\sum_{i=1}^{u}R_i\gamma_i;\upsilon_1,...,\upsilon_r;1), \\ \cdots & \cdots \\ \cdots & B,(1-\lambda-\mu+\alpha-n'\xi-\sum_{i=1}^{u}R_i\gamma_i;\upsilon_1,...,\upsilon_r;1), \\ z_r & \end{array}\right.$$

$$\left.\begin{array}{l}(1-\lambda-\mu+\alpha+\beta-n'\xi-\sum_{i=1}^{u}R_i\gamma_i;\upsilon_1,...,\upsilon_r;1):C \\ \\ (1-\lambda-\mu+\beta-n'\xi-\sum_{i=1}^{u}R_i\gamma_i;\upsilon_1,...,\upsilon_r;1):D\end{array}\right) \tag{8.3}$$

under the same conditions and notations that (6.1) and

$$B'_u = \frac{(-L)_{h_1R_1+...+h_uR_u}B(E;R_1,...,R_u)}{R_1!,...,R_u!};\ B(L;R_1,...,R_u)$$

is defined by (8.1).

9. Conclusion

In this paper we have evaluated a general Eulerian integral involving the multivariable I-function defined by Nambisan et al [2], a class of polynomials of several variables and the generalized incomplete hypergeometric function. The integral established in this paper is of very general nature as it contains multivariable I-function, which is a general function of several variables studied so far. Thus, the integral established in this research work would serve as a key formula from which, upon specializing the parameters, as many as desired results involving the special functions of one and several variables can be obtained.

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