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ON ENTIRE FUNCTIONS OF SEVERAL VARIABLES WITH DERIVATIVES OF EVEN ORDER TAKING INTEGER VALUES

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Abstract: We extend to several variables an earlier result of ours, according to which an entire function of one variable of sufficiently small exponential type, having all derivatives of even order taking integer values at two points, is a polynomial. The proof in the one dimensional case relies on Lidstone expansion of the function. For n variables, we need $n+1$ points, having the property that the differences of n of them with the remaining one give a basis of $\mathbb{C}^n$. The proof is by reduction to the one variable situation.

Keywords and Phrases: Integer valued entire functions, Lidstone polynomials, exponential type, Pólya's Theorem, analytic functions of several variables.

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1. The Main Result

We denote by $\mathbb{N}$ the set $\{0, 1, 2, \dots\}$. For $\underline{z} = (z_1, \dots, z_n) \in \mathbb{C}^n$ and $\underline{t} = (t_1, \dots, t_n) \in \mathbb{N}^n$, write

$$\underline{z}^{\underline{t}} = z_1^{t_1} \cdots z_n^{t_n}, \quad |\underline{z}| = \max_{1 \le i \le n} |z_i|, \quad \|\underline{t}\| = t_1 + \cdots + t_n, \quad \underline{t}! = t_1! \cdots t_n!$$

and

$$D^{\underline{t}} = \left(\frac{\partial}{\partial z_1}\right)^{t_1} \cdots \left(\frac{\partial}{\partial z_n}\right)^{t_n}.$$