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EXPLICIT EVALUATION OF RATIOS OF THETA FUNCTIONS

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Abstract: In the literature one can find evaluation of ratios of theta function $\frac{f(-q)}{q^{\frac{n-1}{24}}f(-q^n)}$ for $n = 2, 4, 5, 7, 9, 25$. The purpose of this article is to obtain evaluation of $\frac{f(-q)}{q^{\frac{6}{24}}f(-q^6)}$ for certain rational k with $q = e^{-2\pi\sqrt{k}}$.

Keywords and Phrases: Theta functions, Continued fraction.

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1. Introduction

For any complex numbers a and q with $|q| < 1$, we define

$$(a; q)_\infty = \prod_{n=0}^{\infty}(1 - aq^n).$$

Ramanujan general theta-function $f(a, b)$, [6, p. 197], is defined by

$$f(a, b) = \sum_{n=-\infty}^{\infty} a^{\frac{n(n+1)}{2}} b^{\frac{n(n-1)}{2}} = (-a; ab)_\infty(-b; ab)_\infty(ab, ab)_\infty, \quad |ab| < 1. \tag{1.1}$$

He also defines [6, p. 197],

$$f(-q) = f(-q, -q^2) = \sum_{k=-\infty}^{\infty}(-1)^k q^{\frac{k(3k-1)}{2}} = (q; q)_\infty. \tag{1.2}$$