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Modular Relations Involving Eisenstein Series

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Abstract : In this paper, we establish several *P—Q* modular relations for the ratios involving Eisenstein series analogous to those recorded by Ramanujan in his Notebooks. We also introduce a parametrization j_s,t for the series involving Eisenstein series and establish several properties of j_s,t- As an application, we evaluate several explicit values for the Eisenstein series.

Keywords : Eisenstein series; Modular equations.
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1. Introduction

The Eisenstein series in terms of $L(q)$, $M(q)$ and $N(q)$ are defined by

$$L(q) := 1 - 24\sum_{n=1}^{\infty}\frac{nq^n}{1-q^n} \tag{1.1}$$

$$M(q) := 1 + 240\sum_{n=1}^{\infty}\frac{n^3q^n}{1-q^n} \tag{1.2}$$

and

$$N(q) := 1 - 504\sum_{n=1}^{\infty}\frac{n^5q^n}{1-q^n}. \tag{1.3}$$

where $|q| < 1$. On pages 44, 50, 51 and 53 in his Lost Notebook [16], Ramanujan recorded twelve formulas for Eisenstein series. Most of them are representations for certain Eisenstein series in terms of coefficients of Dedekind eta functions, or more precisely, Hauptmodulus. S. Raghavan and S. S. Rangachari [15] have obtained the proofs for all of Ramanujan's identities of Eisenstein series by employing the theory of modular forms, B. C. Berndt et al. [5] have proved Eisenstein identities using the theorems found in Ramanujan Notebooks.

We define Ramanujan theta-function $f(a,b)$ [1] as follows:

$$f(a,b) = \sum_{n=-\infty}^{\infty} a^{n(n+1)/2}b^{n(n-1)/2}, \quad \text{where } |ab| < 1. \tag{1.4}$$

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