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RAMANUJAN'S EISENSTEIN SERIES OF LEVEL 3 AND 6, ITS APPLICATION

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Abstract: In this paper, we give an elementary proof of Eisenstein series of level 3 and 6, recorded by Ramanujan. Further, as an application of these Eisenstein series identities, we establish certain incomplete elliptic integrals of the first kind, involving theta-functions, which are analogous to integrals recorded by Ramanujan in his Lost Notebook.

Keywords and Phrases: Dedekind eta-function, Theta-functions, Eisenstein series, Bilateral basic hypergeometric series, Incomplete elliptic integrals.

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1. Introduction

Let $P(q)$ denote the Eisenstein series of weight 2, defined by

$$P(q) := 1 - 24\sum_{k=1}^{\infty}\frac{kq^k}{1-q^k}. \tag{1.1}$$

For any positive integer n, let P_n be defined by

$$P_n := P(q^n).$$