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VALUE DISTRIBUTION OF L-FUNCTIONS AND MEROMORPHIC FUNCTIONS SHARING FINITE SETS

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Abstract: Let f be a non-constant meromorphic function with finitely many poles, and let L be an L-function in the Selberg class. In ([13]) the authors showed the existence of subsets $S, T \subset \mathbb{C}$ of 10 elements such that the condition $L^{-1}(S) = f^{-1}(T)$ implies $f = hL$ for a non-zero constant h. In this paper, we present a class of such subsets $S, T \subset \mathbb{C}$ of 9 elements. As an application of this result, we obtain a class of subsets $S \subset \mathbb{C}$ of 9 elements such that the condition $L^{-1}(S) = f^{-1}(S)$ implies $f = L$. This result improves ([22], Theorem 7).

Keywords and Phrases: L-function, Selberg class, value distribution, uniqueness, meromorphic functions.

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1. Introduction. Main results

L-functions in the Selberg class, with the Riemann zeta function as a prototype, are important objects in number theory. In this paper, an L-function always means a non-constant L-function in the Selberg class $\mathcal{S}$, with the normalized condition $a(1) = 1$, which is defined to be a Dirichlet series

$$L(s) = \sum_{i=0}^{\infty} \frac{a(n)}{n^s}$$