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ON STRONGLY MULTIPLICATIVE GRAPHS

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Abstract : In this note we show that every wheel is strongly multiplicative. Also we give a formula for $\lambda(n)$, the maximum number of edges in a strongly multiplicative graph of order n.

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1. Introduction

In an interesting paper [2] L. W. Beineke and S. M. Hegde have studied strongly multiplicative graphs. In fact they have shown that all graphs like trees, wheels and grids are strongly multiplicative. They have also obtained an upper bound for the maximum number of edges $\lambda(n)$ for a given strongly multiplicative graph of order n. Recently in [1], C. Adiga, H. N. Ramaswamy and D. D. Somashekara have obtained a sharper upper bound for $\lambda(n)$.

In this note we give an alternate proof using Bertrand's hypothesis, of the result that every wheel is strongly multiplicative. Erdös [3] has obtained an asymptotic formula for $\lambda(n)$. In his lecture at the International Conference on Discrete Mathematics and Number Theory, held at Tiruchirapalli, Tamil Nadu, India, L. W. Beineke had posed the problem of obtaining exact formula for $\lambda(n)$. In this note we obtain a formula for $\lambda(n)$ in terms of divisor functions.

We recall the definition of strongly multiplicative graphs given in [2].

Definition. A graph with n vertices is said to be strongly multiplicative if its vertices can be labelled $1, 2, \cdots, n$ so that the values on the edges, obtained as the product of labels of their end vertices, are all distinct.

We give a simple proof of the result in [2] that every wheel is strongly multiplicative. A wheel $W_{n+1} (n \geq 3)$ is a graph with $n+1$ vertices: $v_1, v_2, \cdots v_n, w$, such that $[v_1, v_2, \cdots, v_n, v_1]$ is a cycle $[v_i, w]$ is an edge for $1 \leq i \leq n$.