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GROWTH PROPERTIES OF A CLASS OF ENTIRE DOUBLE DIRCHLET SEQUENCES

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Abstract: In this paper, we have defined the growth (order and type) of complex double sequences analogous to the respective definitions in the theory of integral functions represented by double Dirichlet series with fixed sequences $\{\lambda_m\}$ and $\{\mu_n\}$ of exponents. We have considered a class P of entire double Dirchlet sequences (EDDS) and have examined their growth properties. Distribution of elements of P over the universal set U of all EDDS has been investigated and has been depicted by Venn diagram.

1. Introduction

Let $\{\lambda_m\}$ and $\{\mu_n\}$ be strictly increasing and divergent sequence of positive reals satisfying

$$\lim_{m\to\infty} \frac{\log m}{\lambda_m} \;=\; 0 \;=\; \lim_{n\to\infty} \frac{\log n}{\mu_n} \tag{1.1}$$

A double Dirichlet series $f(s_1, s_2) = \sum a_{mn}\, e^{s_1\lambda_m + s_2\mu_n}$ represents an entire function if

$$\lim_{m+n\to\infty} \frac{\log |a_{mn}|}{\lambda_m + \mu_n} \;=\; -\infty \tag{1.2}$$

Throughout this paper, any complex sequence satisfying (1.2) will be called an entire double Dirichlet sequence (EDDS, in short).

The Ritt order, or simply order, $\rho(f)$ of the entire Dirichlet series $f(s) = \sum\, a_n e^{s\lambda_n}$ is defined as

$$\rho(f) \;=\; \lim_{\sigma\to\infty} \sup\, \frac{\log\log M(\sigma, f)}{\log \sigma}$$

where

$$M(\sigma, f) \;=\; \overset{l.u.b.}{-\infty < t < \infty}\, |f(\sigma + it)|$$