South East Asian J. Math. & Math. Sc.
Vol.6 No.2(2008), pp.69–88

A THEOREM CHARACTERIZING KEY HYPERGEOMETRIC TRANSFORMATIONS IN RAMANUJAN'S DEVELOPMENT OF CLASSICAL AND ALTERNATIVE THEORIES OF ELLIPTIC FUNCTIONS[1]

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(*Received: Februray 20, 2008*)

Dedicated to Professor G.E. Andrews on his seventieth birthday

1. Introduction

In his famous paper on modular equations and approximations to π, Ramanujan [2] remarks that "There are corresponding theories in which q is replaced by one or the other of the functions

$$q_3 := \exp\Big(-\frac{2\pi}{\sqrt{3}}\cdot\frac{K_3'}{K_3}\Big),\quad q_4 := \exp\Big(-\frac{\pi\sqrt{2}K_4'}{K_4}\Big)$$

and

$$q_6 := \exp\Big(-\frac{2\pi K_6'}{K_6}\Big),$$

where

$$K_3 := K_3(k) := {}_2F_1 Big(\frac{1}{3},\frac{2}{3};1;k\Big)\quad K_4 := K_4(k) := {}_2F_1\Big(\frac{1}{4},\frac{3}{4};1;k\Big)$$

and

$$K_6 := K_6(k) := {}_2F_1\Big(\frac{1}{6},\frac{5}{6};1;k\Big)."$$

Here K_j' stands for $K_j(k')$ with $k' = \sqrt{1-k^2}$ and q stands for the classical base given by $q = q_2 := \exp\Big(-\frac{\pi K'}{K}\Big)$ with $K := K(k) := {}_2F_1\Big(\frac{1}{2},\frac{1}{2};1;k\Big)$. In fact on six pages, pp. 257-262, of his second notebook [3], Ramanujan gives approximately 50 results without proofs in these theories. All these results have been proved by Berndt, Bhargava and Garvan in [1], where the theory based on q_r is called theory to base q_r or theory to signature r. Among these results are the following hypergeometric transformations (1.1)-(1.8), which play a key role

[1]Supported under DST Project no. DST/MS/059/96.