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ON AN EXAMPLE RELATED TO A CONJECTURE OF RAJENDRA BHATIA AND PETER ŠEMRL

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Dedicated to Professor G.E. Andrews on his seventieth birthday

Abstract: Suppose A and B are two linear operators on C^n with a non-Euclidean norm. In a paper [1] on orthogonality of matrices Bhatia and Šemrl conjectures that $\|A\| \leq \|A + \lambda B\|$ for all $\lambda \in C$ iff there exists a unit vector $\tilde{z} \in C^n$ such that $\|A\tilde{z}\| = \|A\|$ and $\|A\tilde{z}\| \leq \|(A + \lambda B)\tilde{z}\|$ for all $\lambda \in C$. The conjecture was negated by Li [2]. We here give an easy example to negate a slightly modified form of the the conjecture $\|A\| < \|A+\lambda B\|$ for all non-zero scalar $\lambda \in C$ iff there exists a unit vector $\tilde{z} \in C^n$ such that $\|A\tilde{z}\| = \|A\|$ and $\|A\tilde{z}\| < \|(A + \lambda B)\tilde{z}\|$ for all non-zero scalar $\lambda \in C$.

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1. Introduction

Clearly one part of the modified form of the conjecture is always true i.e., if there exists a unit vector $\tilde{z} \in C^n$ such that $\|A\tilde{z}\| = \|A\|$ and $\|A\tilde{z}\| < \|(A + \lambda B)\tilde{z}\|$for all non-zero scalar $\lambda \in C$ then $\|A\| < \|A + \lambda B\|$ for all non-zero scalar $\lambda \in C$. In fact this part is true for the original conjecture. We give an example to show that the other part is not always true.

Consider C^n with the norm $\|.\|_\infty$ defined as $\|\tilde{z}\|_\infty = \max\{|z_1|, |z_2| \ldots |z_n|\}$, where

$$\tilde{z} = \begin{pmatrix} z_1 \\ z_2 \\ . \\ z_n \end{pmatrix} \in C^n$$