

ON A LINEAR FORM FOR CATALAN'S CONSTANT

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Dedicated to George Andrews

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Abstract: It is shown how Andrews' multidimensional extension of Watson's transformation between a very-well-poised ${}_8\phi_7$ -series and a balanced ${}_4\phi_3$ -series can be used to give a straightforward proof of a conjecture of Zudilin and the second author on the arithmetic behaviour of the coefficients of certain linear forms of 1 and Catalan's constant. This proof is considerably simpler and more stream-lined than the first proof, due to the second author.

Keywords and Phrases: Catalan's constant, linear forms, hypergeometric series, Andrews' identity

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1. Introduction

Andrews' multidimensional extension [1, Theorem 4] of Watson's transformation between a very-well-poised ${}_8\phi_7$ -series and a balanced ${}_4\phi_3$ -series [6, (2.5.1); Appendix (III.18)] in its full beauty reads

$$\begin{aligned} & \sum_{k=0}^n \frac{(a; q)_k (q\sqrt{a}; q)_k (-q\sqrt{a}; q)_k (b_1; q)_k (c_1; q)_k \cdots (b_{m+1}; q)_k (c_{m+1}; q)_k (q^{-n}; q)_k}{(\sqrt{a}; q)_k (-\sqrt{a}; q)_k (qa/b_1; q)_k (qa/c_1; q)_k \cdots (qa/b_{m+1}; q)_k (qa/c_{m+1}; q)_k (q^{n+1}a; q)_k} \\ & \quad \times \left(\frac{a^{m+1} q^{m+1+n}}{b_1 c_1 \cdots b_{m+1} c_{m+1}} \right)^k \\ & = \frac{(qa; q)_n (qa/b_{m+1} c_{m+1}; q)_n}{(qa/b_{m+1}; q)_n (qa/c_{m+1}; q)_n} \sum_{0 \leq i_1 \leq i_2 \leq \cdots \leq i_m \leq n} \frac{a^{i_1 + \cdots + i_{m-1}} q^{i_1 + \cdots + i_m}}{(b_2 c_2)^{i_1} \cdots (b_m c_m)^{i_{m-1}}} \end{aligned}$$