South East Asian J. Math. & Math. Sc.
Vol.6 No.2(2008), pp.57–61

ON A THREE VARIABLE RECIPROCITY THEOREM

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(*Received: September 12, 2007*)

Dedicated to Professor G. E. Andrews on his seventieth birthday

Abstract: In this note, we give a proof of three variable reciprocity theorem using q-binomial theorem and Gauss summation formula.

Keywords and Phrases: q-binomial theorem, q-series, three variable, reciprocity theorem
2000 AMS Subject Classification: 33D15, 11B65

1. Introduction

In his "lost" notebook [11] Ramanujan stated several results related to q-series and one of them is the following beautiful reciprocity theorem:

$$\rho(a,b)-\rho(b,a) = \left(\frac{1}{b}-\frac{1}{a}\right)\frac{(aq/b)_\infty\,(bq/a)_\infty\,(q)_\infty}{(-aq)_\infty\,(-bq)_\infty} \tag{1.1}$$

where

$$\rho(a,b) := \left(1+\frac{1}{b}\right)\sum_{n=0}^{\infty}\frac{(-1)^n\,q^{n(n+1)/2}\,a^n\,b^{-n}}{(-aq)_n}, \quad a\neq -q^{-n} \quad \text{and} \quad |q|<1,$$

$$(a)_0 := (a;q)_0 = 1,$$

$$(a)_\infty := (a;q)_\infty = \prod_{n=0}^{\infty}(1-aq^n)$$

and

$$(a)_n := (a;q)_n = \frac{(a;q)_\infty}{(aq^n;q)_\infty}, \quad -\infty<n<\infty.$$

The first proof of (1.1) was given by Andrews [2] using four free-variable identity and Jacobi's triple product identity. Further, in his paper [3], Andrews