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ON THE ORDINARY AND SIGNED GÖLLNITZ-GORDON PARTITIONS

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Dedicated to George Andrews on the occasion of his 70th birthday

Abstract: In [Bull. Amer. Math. Soc. 44 (2007) 561–573], George Andrews introduced the concept of a "signed partition," i.e. a representation of a positive integer as an unordered sum of integers, some possibly negative. In that paper, Andrews provides an alternate combinatorial interpretation, in terms of signed partitions, of a certain q-series identity associated with the Göllnitz-Gordon partition theorem. In this paper, I present a bijection between the "ordinary" and "signed" Göllnitz-Gordon partitions.

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1. Introduction

A partition of an integer n is a representation of n as an unordered sum of positive integers. In a recent paper [1], Andrews introduced the notion of a "signed partition," that is, a representation of a positive integer as an unordered sum of integers, some possibly negative.

Consider the following q-series identity:

Theorem 1.1 (Ramanujan and Slater). For $|q| < 1$,

$$\sum_{j=0}^{\infty} \frac{q^{j^2}(1+q)(1+q^3)\dots(1+q^{2j-1})}{(1-q^2)(1-q^4)\dots(1-q^{2j})} = \prod_{\substack{m\geqq 1 \\ m\equiv 1,4,7 \pmod 8}} \frac{1}{1-q^m}. \tag{1.1}$$

An identity equivalent to (1.1) was recorded by Ramanujan in his lost notebook [2, Entry 1.7.11]. The first proof of (1.1) was given by Slater [5, Eq.(36)].

Identity (1.1) became well known after Gordon [4] showed that it is equivalent to the following partition identity, which had been discovered independently by Göllnitz [3]: