

IMPULSIVE CONTROL IN FUZZY NEUTRAL SECOND-ORDER FUNCTIONAL DIFFERENTIAL EQUATIONS

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Abstract: This paper investigates the controllability of impulsive second-order neutral fuzzy functional differential systems with delay. Using the Banach fixed-point theorem and the framework of fuzzy numbers represented through their α -level sets, we derive sufficient conditions for controllability and for the existence and uniqueness of fuzzy mild solutions. An explicit admissible control is constructed

through the inverse of the α -level controllability operator. The contraction condition incorporates the Lipschitz constants of the neutral and nonlinear terms, the impulse contribution, and a uniform bound for the inverse controllability operator. Two illustrative examples demonstrate how the assumptions and terminal-state controllability criterion can be verified. The proposed framework provides a unified analytical approach for impulsive neutral systems affected by uncertainty, delay, and abrupt state changes.

Keywords and Phrases: Controllability, Mild solution, Fixed point, Fuzzy differential equation, Contraction mapping, Neutral system, Impulsive system, Delay.

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1. Introduction

Differential equations provide a fundamental framework for modeling dynamic phenomena in science and engineering. Since real-world systems frequently involve vague, incomplete, or imprecise information, fuzzy set theory provides a useful framework for incorporating uncertainty into mathematical models. Fuzzy differential equations (FDEs) have consequently received considerable attention; see [2, 3, 6, 9, 11, 20].

Several numerical approaches have also been developed for fuzzy differential and integral equations. Narayanamoorthy and Thangapandi [16] used a generalized Runge–Kutta method for systems of fuzzy differential equations of integer and fractional order. Narayanamoorthy and Sathiyapriya [14] proposed a homotopy perturbation approach for nonlinear fuzzy integro-differential equations, while Sathiyapriya and Narayanamoorthy [22] developed a numerical method for linear two-dimensional fuzzy integral equations. Narayanamoorthy and Mathankumar [15] considered hybrid fuzzy differential equations using Picard’s method, and Vinothini and Kavitha [23] compared numerical methods for fuzzy delay differential equations. These studies demonstrate the importance of α -level representations and numerical methods for uncertain differential systems, but they do not address the controllability of the impulsive second-order neutral fuzzy system considered here.

Fuzzy techniques have also been applied in other uncertainty-management settings. Prabu and Narayanamoorthy [18] developed an integrated hybrid segmentation method using fuzzy clustering for brain-tumor image segmentation. More recently, similarity and aggregation methods have been developed for T-spherical, q-rung orthopair, and complex hesitant fuzzy environments [4, 8, 13, 19]. Although these studies demonstrate the broad applicability of fuzzy methodologies,

their objectives and mathematical frameworks differ from the controllability problem addressed in this paper.

Neutral functional differential equations (NFDEs), which involve current and delayed states, are widely used to model systems with memory effects in physics, engineering, and biology [12]. Impulsive differential equations describe abrupt changes at prescribed instants and arise in population dynamics, pharmacokinetics, electrical circuits, and economic systems [5, 10]. Controllability, which concerns steering a system to a prescribed terminal state through a suitable control, is fundamental in control theory and has been studied for nonlinear systems with and without impulses [17, 21].

Despite these developments, a clear gap remains. Existing fuzzy differential-equation studies mainly focus on existence, approximation, numerical solution, or related fuzzy decision methodologies, whereas controllability of second-order neutral fuzzy systems simultaneously incorporating delay and impulsive effects has received comparatively limited attention. In particular, the explicit construction of an admissible fuzzy control through an α -level controllability operator, together with a contraction condition that accounts for the inverse controllability operator, has not been sufficiently developed in the framework considered here. Chalishajar and Ramesh [7] studied impulsive fuzzy solutions for abstract second-order partial neutral functional differential equations, but did not establish the present controllability criterion with explicit control construction. Acharya et al. [1] investigated controllability of neutral impulsive functional differential equations with nonlocal conditions, but their framework does not cover the present second-order abstract fuzzy system with delay.

Motivated by this gap, the research question addressed in this paper is: under what verifiable conditions can an impulsive second-order neutral fuzzy functional differential system with delay be driven from an arbitrary initial state to a prescribed terminal fuzzy state? To answer this question, we formulate the associated mild solution, construct an admissible control using the inverse α -level controllability operators, and apply the Banach fixed-point theorem.

The main contributions are:

1. We establish a controllability criterion for impulsive second-order neutral fuzzy functional differential systems with delay, extending the framework of [7].
2. We employ the Banach fixed-point theorem to establish existence and uniqueness of fuzzy mild solutions under an explicit contraction condition.

3. We construct admissible control functions using the α -level set approach under the bijectivity and uniform boundedness of the controllability operators G_l^α and G_r^α .
4. We provide illustrative examples showing how the Lipschitz and contraction conditions can be checked for particular fuzzy systems.

The paper is organized as follows. Section 2 presents the required fuzzy concepts and metric spaces. Section 3 formulates the problem, states the hypotheses, constructs the control, and establishes controllability by the Banach fixed-point theorem. Section 4 presents illustrative examples, and Section 5 concludes the paper with the principal findings, limitations, applications, and future directions.

2. Preliminaries

This section introduces the basic notation, definitions, and preliminary results on fuzzy numbers used throughout the manuscript. A fuzzy number extends the concept of a crisp number by incorporating uncertainty through a membership function taking values in $[0, 1]$.

Let $CC(\mathbb{R}^n)$ denote the family of all non-empty, convex, and compact subsets of $\mathbb{R}^n$. We define $E^n = \{q : \mathbb{R}^n \rightarrow [0, 1] \mid q \text{ is upper semi-continuous, fuzzy convex, compact, and normal}\}$

Definition 2.1. [20] *The Hausdorff metric $d_H : CC(\mathbb{R}^n) \times CC(\mathbb{R}^n) \rightarrow \mathbb{R}_+$ is defined by*

$$d_H(M, N) = \max \left\{ \sup_{m \in M} \inf_{n \in N} \|m - n\|, \sup_{n \in N} \inf_{m \in M} \|n - m\| \right\}.$$

Then $(CC(\mathbb{R}^n), d_H)$ is a complete metric space.

Definition 2.2. [20] *The supremum metric d_∞ on E^n is defined by*

$$d_\infty(w, v) = \sup_{0 \leq \alpha \leq 1} d_H([w]^\alpha, [v]^\alpha), \quad w, v \in E^n.$$

Then (E^n, d_∞) is a complete metric space.

Let $\tau > 0$ be a fixed delay. For functions with a prescribed history on $[-\tau, 0]$, define

$$\Omega' = \{p \in C([-\tau, T], E^n) : p(t) = \phi(t) \text{ for } t \in [-\tau, 0]\}$$

and

$$H_1(p, q) = \sup_{-\tau \leq t \leq T} d_\infty(p(t), q(t)).$$

The space (Ω', H_1) is complete.

Definition 2.3. [20] For a fuzzy process $y(t)$, its derivative, whenever it exists as a fuzzy process, is represented at level α by

$$[y'(t)]^\alpha = [(y_l^\alpha)'(t), (y_r^\alpha)'(t)].$$

Definition 2.4. [20] The fuzzy integral is defined by

$$\left[\int_c^d y(t) dt \right]^\alpha = \left[\int_c^d y_l^\alpha(t) dt, \int_c^d y_r^\alpha(t) dt \right],$$

provided the integrals on the right-hand side exist.

Definition 2.5. [7] A system is controllable on $J = [0, T]$ if, for every initial state and every prescribed terminal state $p^1 \in E^n$, there exists an admissible control u such that the corresponding fuzzy mild solution satisfies $p(T) = p^1$.

Notation

E^n	Space of fuzzy numbers in $\mathbb{R}^n$
d_H	Hausdorff metric
d_∞	Supremum metric on E^n
H_1	Metric on the fuzzy-valued function space
$R(t)$	Second-order resolvent family
$C(t)$	Associated cosine family
G	Controllability operator
G_l^α, G_r^α	α -level controllability operators
M_G	Uniform bound for the inverse controllability operators
d_g	Lipschitz constant of the neutral term
d_f	Lipschitz constant of the nonlinear term
d_i	Lipschitz constant of the impulse term
N	Number of impulse points

List of Abbreviations

FDE	Fuzzy Differential Equation
NFDE	Neutral Functional Differential Equation

3. Controllability Results

3.1. Problem Formulation

Consider the impulsive second-order neutral fuzzy functional differential system

$$\begin{cases} D^2[p(t) - r(t, p_t)] = Ap(t) + F(t, p_t) + Bu(t), & t \in J \setminus \{t_1, \dots, t_N\}, \\ p(0) = \xi, \\ D[p(t) - r(t, p_t)]|_{t=0} = z, \\ \Delta p(t_i) = I'_i(p(t_i^-)), \end{cases} \quad i = 1, \dots, N, \quad (1)$$

where $J = [0, T]$, $p_t(\theta) = p(t + \theta)$ for $\theta \in [-\tau, 0]$, $r, F : J \times C([-\tau, 0], E^n) \rightarrow E^n$ are nonlinear mappings, $\xi \in E^n$ is the initial state, $z \in E^n$ is the initial derivative of the neutral component, $I'_i \in C(E^n, E^n)$ are impulse mappings, and $u : J \rightarrow E^n$ is an admissible control. The symbols A and B denote the corresponding bounded linear operators in the mild-solution representation.

Let $R(t)$ and $C(t)$ denote the fuzzy resolvent families associated with the second-order linear part. We assume that the required products and integrals are well defined at every α -level.

3.2. Hypotheses and Assumptions

(H1) Let $[R(t)]^\alpha = [R_l^\alpha(t), R_r^\alpha(t)]$ with $R(0) = I$, and assume

$$|R_j^\alpha(t)| \leq m_1, \quad |AR_j^\alpha(t)| \leq m_0, \quad j = l, r,$$

for some $m_1 > 0$, $m_0 \geq 0$, and all $t \in J$ and $\alpha \in [0, 1]$.

(H2) Let $[C(t)]^\alpha = [C_l^\alpha(t), C_r^\alpha(t)]$ with $C(0) = I$ and assume

$$|C_j^\alpha(t)| \leq m_2, \quad j = l, r,$$

for some $m_2 > 0$, all $t \in J$, and all $\alpha \in [0, 1]$.

(H3) There exist positive constants $d_g, d_f > 0$ such that r and F are strongly measurable in t and satisfy

$$d_H([r(t, p_t)]^\alpha, [r(t, q_t)]^\alpha) \leq d_g d_H([p(t)]^\alpha, [q(t)]^\alpha),$$

$$d_H([F(t, p_t)]^\alpha, [F(t, q_t)]^\alpha) \leq d_f d_H([p(t)]^\alpha, [q(t)]^\alpha)$$

for all $t \in J$, $\alpha \in [0, 1]$, and $p, q \in \Omega'$.

(H4) There exists $d_i > 0$ such that, for each impulse time t_i ,

$$d_H([I'_i(p(t_i^-))]^\alpha, [I'_i(q(t_i^-))]^\alpha) \leq d_i d_H([p(t_i^-)]^\alpha, [q(t_i^-)]^\alpha).$$

(H5) Let

$$M_G = \sup_{\alpha \in [0, 1]} \max \{ \|(G_l^\alpha)^{-1}\|, \|(G_r^\alpha)^{-1}\| \} < \infty.$$

Let N denote the finite number of impulse points in J . Assume that

$$[1 + Tm_1\|B\|M_G] [d_g + T(m_0d_g + m_1d_f) + Nm_2d_i] < 1.$$

(H6) For each $\alpha \in [0, 1]$, the α -level controllability operators G_l^α and G_r^α are bijective.

3.3. Mild Solution and Controllability

Definition 3.1. A function $p \in \Omega'$ is called a mild solution of system (1) if it satisfies

$$\begin{aligned} p(t) = & C(t)(\xi - r(0, \xi)) + R(t)z + r(t, p_t) \\ & + \int_0^t AR(t-s)r(s, p_s) ds + \int_0^t R(t-s)[F(s, p_s) + Bu(s)] ds \\ & + \sum_{0 < t_i < t} C(t-t_i)I'_i(p(t_i^-)), \quad t \in J. \end{aligned} \quad (2)$$

Definition 3.2. System (1) is controllable on J if for every $p^1 \in E^n$ there exists an admissible control u such that the mild solution of (2) satisfies

$$p(T) = p^1, \quad [p(T)]^\alpha = [p^1]^\alpha, \quad \alpha \in [0, 1].$$

3.4. Construction of the Control Function

To establish controllability, we define the controllability operator $G : \mathcal{P}_e(\mathbb{R}) \rightarrow E^n$ by

$$G^\alpha(w) = \left[\int_0^T R^\alpha(T-s)Bw(s) ds \right],$$

where w belongs to the admissible control space.

By hypothesis (H6), for each $\alpha \in [0, 1]$, the endpoint operators G_l^α and G_r^α are bijective. We define

$$G_l^\alpha(w_l) = \int_0^T R_l^\alpha(T-s)Bw_l(s) ds,$$

and

$$G_r^\alpha(w_r) = \int_0^T R_r^\alpha(T-s)Bw_r(s) ds.$$

The α -level set of the admissible control is given by

$$[u(s)]^\alpha = [u_l^\alpha(s), u_r^\alpha(s)].$$

For $p \in \Omega'$, define the lower and upper control components by

$$\begin{aligned} u_l^\alpha = (G_l^\alpha)^{-1} & \left[(p^1)_l^\alpha - C_l^\alpha(T)(\xi_l^\alpha(0) - r_l^\alpha(0, \xi)) \right. \\ & - R_l^\alpha(T)z_l^\alpha - r_l^\alpha(T, p_T) - \int_0^T AR_l^\alpha(T-s)r_l^\alpha(s, p_s) ds \\ & - \int_0^T R_l^\alpha(T-s)F_l^\alpha(s, p_s) ds \\ & \left. - \sum_{0 < t_i < T} C_l^\alpha(T-t_i)I'_{il}(p_l^\alpha(t_i^-)) \right], \end{aligned}$$

and

$$\begin{aligned} u_r^\alpha = (G_r^\alpha)^{-1} & \left[(p^1)_r^\alpha - C_r^\alpha(T)(\xi_r^\alpha(0) - r_r^\alpha(0, \xi)) \right. \\ & - R_r^\alpha(T)z_r^\alpha - r_r^\alpha(T, p_T) - \int_0^T AR_r^\alpha(T-s)r_r^\alpha(s, p_s) ds \\ & - \int_0^T R_r^\alpha(T-s)F_r^\alpha(s, p_s) ds \\ & \left. - \sum_{0 < t_i < T} C_r^\alpha(T-t_i)I'_{ir}(p_r^\alpha(t_i^-)) \right]. \end{aligned}$$

Thus, the control is constructed from the inverse of the controllability operator so that the terminal condition

$$[p(T)]^\alpha = [p^1]^\alpha$$

is satisfied.

3.5. Contraction Mapping and Fixed Point

For each $p \in \Omega'$, define

$$\begin{aligned} (\Phi p)(t) = & C(t)(\xi - r(0, \xi)) + R(t)z + r(t, p_t) \\ & + \int_0^t AR(t-s)r(s, p_s) ds + \int_0^t R(t-s)F(s, p_s) ds \\ & + \sum_{0 < t_i < t} C(t-t_i)I'_i(p(t_i^-)) + \int_0^t R(t-s)Bu_p(s) ds. \end{aligned} \tag{3}$$

By the definition of u_p , $(\Phi p)(T) = p^1$. Thus a fixed point of Φ is a mild solution of (1) reaching the prescribed terminal state.

Theorem 1. *If hypotheses (H1)–(H6) are satisfied and the controls constructed above are admissible, then system (1) is controllable on J and admits a unique mild solution associated with the constructed control.*

Proof. Let $p, q \in \Omega'$. We estimate the Hausdorff distance between $(\Phi p)(t)$ and $(\Phi q)(t)$ at an arbitrary $\alpha \in [0, 1]$.

Step 1 (Neutral term): By (H3),

$$d_H([r(t, p_t)]^\alpha, [r(t, q_t)]^\alpha) \leq d_g H_1(p, q).$$

Step 2 (Neutral integral term): By (H1) and (H3),

$$\begin{aligned} & d_H \left(\left[\int_0^t AR(t-s)r(s, p_s) ds \right]^\alpha, \left[\int_0^t AR(t-s)r(s, q_s) ds \right]^\alpha \right) \\ & \leq m_0 d_g \int_0^t d_H([p(s)]^\alpha, [q(s)]^\alpha) ds \\ & \leq T m_0 d_g H_1(p, q). \end{aligned}$$

Step 3 (Nonlinear term): By (H1) and (H3),

$$\begin{aligned} & d_H \left(\left[\int_0^t R(t-s)F(s, p_s) ds \right]^\alpha, \left[\int_0^t R(t-s)F(s, q_s) ds \right]^\alpha \right) \\ & \leq m_1 d_f \int_0^t d_H([p(s)]^\alpha, [q(s)]^\alpha) ds \\ & \leq T m_1 d_f H_1(p, q). \end{aligned}$$

Step 4 (Impulse Term): By (H2) and (H4),

$$\begin{aligned} & d_H \left(\left[\sum_{0 < t_i < t} C(t-t_i)I'_i(p(t_i^-)) \right]^\alpha, \left[\sum_{0 < t_i < t} C(t-t_i)I'_i(q(t_i^-)) \right]^\alpha \right) \\ & \leq m_2 d_i \sum_{0 < t_i < t} d_H([p(t_i^-)]^\alpha, [q(t_i^-)]^\alpha) \\ & \leq N m_2 d_i H_1(p, q), \end{aligned}$$

where N denotes the finite number of impulse points in J .

Step 5 (Control Term): From the definition of the terminal residual and hypotheses (H3)–(H4), we obtain

$$d_H([u_p(s)]^\alpha, [u_q(s)]^\alpha) \leq M_G \left[d_g + T(m_0 d_g + m_1 d_f) + N m_2 d_i \right] H_1(p, q).$$

Consequently, using (H1),

$$\begin{aligned} & d_H \left(\left[\int_0^t R(t-s)Bu_p(s) ds \right]^\alpha, \left[\int_0^t R(t-s)Bu_q(s) ds \right]^\alpha \right) \\ & \leq \int_0^t |R(t-s)| \|B\| d_H([u_p(s)]^\alpha, [u_q(s)]^\alpha) ds \\ & \leq \int_0^t m_1 \|B\| M_G \left[d_g + T(m_0 d_g + m_1 d_f) + Nm_2 d_i \right] H_1(p, q) ds \\ & \leq Tm_1 \|B\| M_G \left[d_g + T(m_0 d_g + m_1 d_f) + Nm_2 d_i \right] H_1(p, q). \end{aligned}$$

Combining Steps 1–5 and taking the supremum over $t \in [0, T]$ and $\alpha \in [0, 1]$, we obtain

$$\begin{aligned} H_1(\Phi(p), \Phi(q)) & \leq [1 + Tm_1 \|B\| M_G] \\ & \quad \times [d_g + T(m_0 d_g + m_1 d_f) + Nm_2 d_i] H_1(p, q). \end{aligned}$$

By hypothesis (H5), there exists a constant $\kappa \in (0, 1)$ such that

$$H_1(\Phi(p), \Phi(q)) \leq \kappa H_1(p, q).$$

Hence, Φ is a contraction mapping on the complete metric space (Ω', H_1) . By the Banach fixed-point theorem, Φ has a unique fixed point $p \in \Omega'$. Since

$$(\Phi p)(T) = p^1,$$

the fixed point satisfies

$$p(T) = p^1.$$

Therefore, the system is controllable on J .

4. Illustrative Examples

Example 1. Consider the scalar impulsive fuzzy neutral system

$$\begin{cases} D^2[x(t) - 2tx(t+h)^2] = 2x(t) + 2tx(t+h)^2 + u(t), & t \in [0, b], \\ x(0) = \xi, \\ D[x(t) - 2tx(t+h)^2]|_{t=0} = z, \\ \Delta x(t_k) = I'_k(x(t_k^-)), & I'_k(x) = \frac{1}{x}, \end{cases} \quad (4)$$

where $h \in [-\tau, 0]$ is a fixed delay and the state is restricted to a bounded positive interval $[a, M]$ with $a > 0$. The latter restriction ensures that the reciprocal impulse map is Lipschitz.

For the triangular fuzzy numbers used below,

$$[9]^\alpha = [\alpha + 8, 10 - \alpha], \quad [2]^\alpha = [\alpha + 1, 3 - \alpha].$$

For $x, y \in [a, M]$,

$$\left| \frac{1}{x} - \frac{1}{y} \right| = \frac{|x - y|}{|xy|} \leq \frac{1}{a^2} |x - y|.$$

Thus the impulse mapping satisfies (H4) with $d_i = a^{-2}$.

For the nonlinear term $f(t, x_t) = 2tx(t + h)^2$, boundedness of the state gives

$$|x^2 - y^2| \leq 2M|x - y|,$$

and hence

$$d_H([f(t, x_t)]^\alpha, [f(t, y_t)]^\alpha) \leq 4bM d_H([x(t)]^\alpha, [y(t)]^\alpha).$$

Thus one may take $d_f = 4bM$ on $J = [0, b]$. The remaining constants m_0, m_1, m_2, M_G are determined by the chosen resolvent families and control operator. If they satisfy (H5), Theorem 1 yields controllability to any prescribed target fuzzy state.

Example 2. Consider the fuzzy second-order system

$$\begin{cases} x_{tt}(t, z) = [\tilde{2}x(t, z)]_{zz} + u(t, z) + \tilde{2}tx(t, z)^2, \\ \Delta x(t_k) = I'_k(x(t_k^-)), \quad k = 1, \dots, N, \\ x(0, z) = \xi(z), \\ x_t(0, z) = z_0(z), \end{cases} \quad (5)$$

with suitable boundary conditions in the spatial variable z . The system is interpreted through the α -level representation and the corresponding second-order resolvent family. For the triangular fuzzy number $\tilde{2}$,

$$[\tilde{2}]^\alpha = [1 + \alpha, 3 - \alpha].$$

If the state is bounded by $|x(t, z)| \leq M$ on the domain, then

$$|x^2 - y^2| \leq 2M|x - y|,$$

so the nonlinear term satisfies a Lipschitz estimate with a finite constant d_f depending on T and M . If the impulse mappings satisfy (H4), the resolvent families satisfy (H1)–(H2), and the associated controllability operators satisfy (H5)–(H6), Theorem 1 guarantees controllability to the prescribed terminal fuzzy state.

4.1. Numerical simulation

To complement Theorem 1 with direct evidence, system (1) was simulated using an Euler-type scheme adapted to fuzzy-valued functions via α -cuts, with $u(t)$ computed via a numerical approximation of G^{-1} so the solution meets the terminal condition. Figure 1 shows (a) fuzzy trajectories $[p(t)]^\alpha$ for $\alpha \in \{0, 0.5, 1\}$ under impulses and delay; (b) the recovered control $u(t)$ for $\alpha = 0.5$; and (c) the convergence of $\|p(t) - p^1\|$ to zero. The impulses at $t = 2, 4$ produce the expected discontinuities, yet the recovered control successfully steers the system to the target fuzzy state, supporting the controllability result established in Theorem 1.

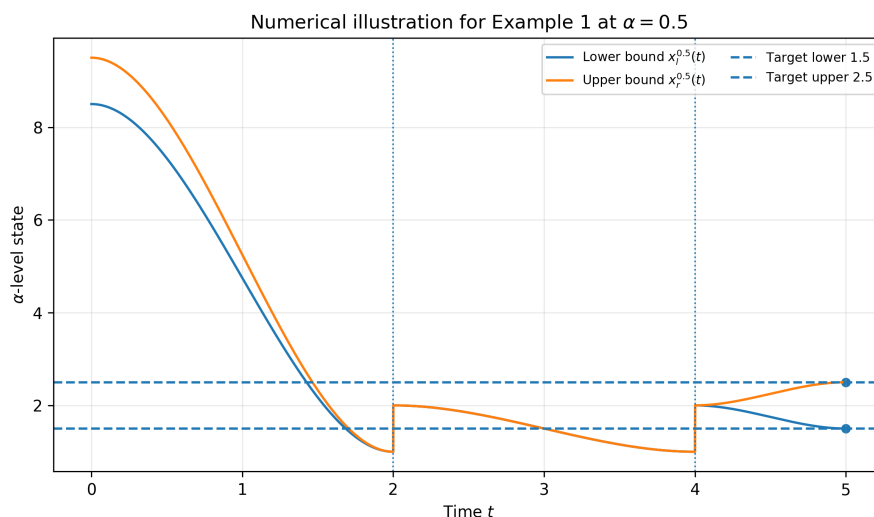


Figure 1: Numerical illustration of the fuzzy state trajectory for $\alpha = 0.5$, with impulse times $t_1 = 2$, $t_2 = 4$, and prescribed terminal state $[p^1]^{0.5} = [1.5, 2.5]$.

5. Conclusion

This paper established sufficient conditions for the controllability of impulsive second-order neutral fuzzy functional differential systems with delay. By constructing an admissible control through the inverse α -level controllability operators and applying the Banach fixed-point theorem, we obtained existence and uniqueness of the corresponding fuzzy mild solution. The contraction constant explicitly combines the Lipschitz properties of the neutral and nonlinear terms, the contribution of the finitely many impulses, and the uniform bound for the inverse controllability operator.

The principal advantage of the approach is that it provides a unified analytical framework for systems in which uncertainty, memory effects, and abrupt state

changes occur simultaneously. The main limitation is the restrictive nature of the contraction condition and the requirement that the controllability operators be bijective with uniformly bounded inverses. These assumptions may limit direct application to systems with strong nonlinearities, large delays, or poorly conditioned control operators.

The illustrative examples show how boundedness and Lipschitz assumptions can be used to verify the hypotheses for particular fuzzy systems. Future work will consider fractional-order impulsive fuzzy neutral systems and weaker fixed-point conditions based on Schauder or Leray–Schauder theorems.

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