

BECK TYPE IDENTITIES FOR n -COLOR PARTITIONS

Shruti Sharma and Amandeep Kaur*

Yadavindra Department of Sciences,
Punjabi University Guru Kashi Campus,
Talwandi Sabo, Punjab, INDIA

E-mail : sharma_stary2k@yahoo.co.in

*Department of Mathematics,
Punjabi University,
Patiala, Punjab, INDIA

E-mail : amandeep.jass7@gmail.com

(**Received:** May 20, 2026 **Accepted:** Aug. 21, 2026 **Published:** Aug. 30, 2026)

Abstract: Euler's identity asserts the equality of the number of partitions into distinct parts and the number of partitions into odd parts of a positive integer m . However, the numbers of parts in these two sets of partitions are not equal, and Beck conjectured that this difference between the number of parts is equal to the number of certain restricted partitions. In this paper, we extend this idea to n -color partitions and obtain Beck type identities for n -color partitions. We obtain two Beck type identities on the n -color analogue of the Euler's identity given by Agarwal. We also provide another Euler type identity for n -color partitions and obtain a Beck type identity for it. In addition to this, we prove the dual form of a Beck type identity for n -color partitions. These dual forms have been introduced by Li and Wang for ordinary partitions.

Keywords and Phrases: n -color partitions, Beck type identities, generating functions.

2020 Mathematics Subject Classification: 05A19, 11P81, 11P84.

1. Introduction

Partition identities are central to the field of partition theory as they uncover significant connections between various partition structures. One of the most celebrated identities due to Euler states that *the number of partitions of a positive integer m into odd parts is the same as the number of partitions of m into distinct parts*. For example, the partitions of 5 into distinct parts are 5, $4 + 1$, $3 + 2$, and partitions into odd parts are 5, $3 + 1 + 1$, $1 + 1 + 1 + 1 + 1$. As asserted by Euler's identity, the numbers of these partitions are equal, but the numbers of parts in these partitions are 5 and 9 respectively. Beck observed that the difference between these numbers of parts is equal to the number of certain restricted partitions. He formulated two conjectures, known as Beck's conjectures, which appeared in the On-Line Encyclopedia of Integer Sequences [10] as entries A090867 and A265251. These conjectures were subsequently proved by Andrews [3] using generating functions, and we state these results as Theorems 1.1 and 1.2.

Theorem 1.1. *The excess of the number of parts in all partitions of a positive integer n into odd parts over the number of parts in all partitions of n into distinct parts equals the number of partitions of n such that the set of even parts has only one element (possibly repeated).*

Theorem 1.2. *The excess of the number of parts in the distinct partitions of a positive integer n over the number of distinct parts in the partitions of n into odd parts equals the number of partitions of n such that there is exactly one part occurring three times while all other parts occur only once.*

Fu-Tang [6] and Yang [11] provided bijective proofs of Theorems 1.1 and 1.2 and also obtained some more generalized results. In recent years, significant developments have been made in this field with many papers focusing on these Beck type identities. For example, see [[4], [5], [7]-[9]].

The development of Beck type identities on Euler's identity motivated us to look for such identities for n -color partitions. These n -color partitions were Introduced by Agarwal and Andrews [2]. *An n -color partition of a positive integer is a partition in which a part of size m can occur in m different colors denoted by subscripts $m_1, m_2, \dots, m_m$.* For example, the n -color partitions of 3 are $3_1, 3_2, 3_3, 2_2 + 1_1, 2_1 + 1_1, 1_1 + 1_1 + 1_1$. The generating function for the number of n -color partitions is

$$\prod_{m=1}^{\infty} \frac{1}{(1 - q^m)^m}.$$

In 2001, Agarwal [1] gave the following n -color analogue of Euler's identity:

$$\prod_{m=1}^{\infty} (1 + q^m)^m = \prod_{m=1}^{\infty} \frac{1}{(1 - q^{2m})^m (1 - q^{2m-1})^{2m-1}}. \quad (1)$$

The combinatorial version of this identity says that *the number of n -color partitions of a positive integer m into distinct parts is equal to the number of n -color partitions of m in which even parts appear with even subscripts only*. In (1), substituting q with q^2 and squaring both sides, we find that the following Euler type identity for n -color partitions also holds:

$$\prod_{m=1}^{\infty} (1 + q^{2m})^{2m} = \prod_{m=1}^{\infty} \frac{1}{(1 - q^{4m})^{2m} (1 - q^{4m-2})^{4m-2}}. \quad (2)$$

The combinatorial version of this identity says that *the number of n -color partitions of a positive integer m with even and distinct parts is equal to the number of n -color partitions of m with even parts where parts divisible by 4 occur with odd subscripts only*.

In Section 2, we obtain two Beck type identities on the n -color analogue of Euler's identity given by (1). We also give a Beck type identity on the Euler type identity given by (2).

In 2021, Li and Wang [7] introduced the dual form of Beck type identities. In the dual form of a Beck type identity, the difference between the cardinalities of two sets of partitions of n turns out to be the same as the number of parts in the partitions of n with certain restrictions. We prove a Beck type identity in the dual form for n -color partitions in Section 3.

2. Beck Type Identities

Throughout this paper, we will use λ to denote an n -color partition; $l(\lambda)$ to denote the length of λ , i.e., the number of parts of λ ; $\bar{l}(\lambda)$ to denote the number of distinct parts of λ and $l_e(\lambda)$ to denote the number of parts of λ that are even with even subscript. We will prove many Beck type identities for different sets of restricted partitions, so we use $A_i(m)$ to denote different sets of n -color partitions with restrictions and $|A_i(m)|$ to denote the cardinality of the set $A_i(m)$ as usual. To prove our first identity, we start by defining the following four sets of restricted n -color partitions:

$A_1(m)$ — The set of n -color partitions of a positive integer m that have exactly one even part with even subscript (possibly repeated) and all the other parts are either even with odd subscripts or parts are odd with any subscript.

$A_2(m)$ – The set of n -color partitions of a positive integer m wherein exactly one part is repeated and all the other parts are distinct.

$A_3(m)$ – The set of n -color partitions of a positive integer m where even parts appear with even subscripts only.

$A_4(m)$ – The set of n -color partitions of a positive integer m into distinct parts.

Theorem 2.1. For all $m \geq 1$,

$$|A_1(m)| = |A_2(m)| = \sum_{\lambda \in A_3(m)} l(\lambda) - \sum_{\lambda \in A_4(m)} l(\lambda).$$

Proof. For the partitions in the set $A_1(m)$, there is exactly one even part which occurs with even subscript. Assuming that this part is $(2s)_{2p}$, it contributes a factor $\frac{sq^{2s}}{1 - q^{2s}}$ as p can take any value from 1 to s . Thus,

$$\begin{aligned} \sum_{m=1}^{\infty} |A_1(m)| q^m &= \left(\sum_{s=1}^{\infty} \frac{sq^{2s}}{1 - q^{2s}} \right) \prod_{k=1}^{\infty} \frac{1}{(1 - q^{2k})^k (1 - q^{2k-1})^{2k-1}} \\ &= \prod_{k=1}^{\infty} (1 + q^k)^k \sum_{m=1}^{\infty} \frac{sq^{2s}}{1 - q^{2s}}. \quad (\text{by (1)}) \end{aligned} \quad (3)$$

Next, for the partitions in the set $A_2(m)$, exactly one part is repeated and all the other parts are distinct. Let us assume that the part s_p occurs at least twice. Then it contributes a factor $\frac{sq^{2s}}{1 - q^s} (1 + q^s)^{s-1}$. Thus,

$$\begin{aligned} \sum_{m=1}^{\infty} |A_2(m)| q^m &= \sum_{s=1}^{\infty} \frac{sq^{2s}}{1 - q^s} (1 + q^s)^{s-1} \prod_{\substack{k=1 \\ k \neq s}}^{\infty} (1 + q^k)^k \\ &= \prod_{k=1}^{\infty} (1 + q^k)^k \sum_{s=1}^{\infty} \frac{sq^{2s}}{1 - q^{2s}}. \end{aligned} \quad (4)$$

Now,

$$\begin{aligned} \sum_{m=1}^{\infty} \left(\sum_{\lambda \in A_3(m)} l(\lambda) \right) q^m \\ = \frac{\partial}{\partial x} \Big|_{x=1} \left(\prod_{k=1}^{\infty} \frac{1}{(1 - xq^{2k})^k (1 - xq^{2k-1})^{2k-1}} \right) \end{aligned}$$

$$\begin{aligned}
&= \prod_{k=1}^{\infty} \frac{1}{(1 - q^{2k})^k (1 - q^{2k-1})^{2k-1}} \left(\sum_{s=1}^{\infty} \frac{(2s-1)q^{2s-1}}{1 - q^{2s-1}} + \sum_{s=1}^{\infty} \frac{sq^{2s}}{1 - q^{2s}} \right) \\
&= \prod_{k=1}^{\infty} (1 + q^k)^k \left(\sum_{s=1}^{\infty} \frac{(2s-1)q^{2s-1}}{1 - q^{2s-1}} + \sum_{s=1}^{\infty} \frac{sq^{2s}}{1 - q^{2s}} \right). \quad (by (1)) \quad (5)
\end{aligned}$$

Similarly,

$$\begin{aligned}
\sum_{m=1}^{\infty} \left(\sum_{\lambda \in A_4(m)} l(\lambda) \right) q^m &= \frac{\partial}{\partial x} \Big|_{x=1} \left(\prod_{k=1}^{\infty} (1 + xq^k)^k \right) \\
&= \prod_{k=1}^{\infty} (1 + q^k)^k \sum_{s=1}^{\infty} \frac{sq^s}{1 + q^s}. \quad (6)
\end{aligned}$$

Also, we have the following result from the sequence A000593 [10]:

$$\sum_{s=1}^{\infty} \frac{sq^s}{1 + q^s} = \sum_{s=1}^{\infty} \frac{(2s-1)q^{2s-1}}{1 - q^{2s-1}}. \quad (7)$$

Proof of identity (7) is as follows:

$$\begin{aligned}
\sum_{s=1}^{\infty} \frac{sq^s}{1 + q^s} &= \sum_{s=1}^{\infty} \frac{sq^s(1 - q^s)}{1 - q^{2s}} \\
&= \left(\sum_{s=1}^{\infty} \frac{sq^s}{1 - q^{2s}} + \sum_{s=1}^{\infty} \frac{sq^{2s}}{1 - q^{2s}} \right) - \sum_{s=1}^{\infty} \frac{2sq^{2s}}{1 - q^{2s}} \\
&= \sum_{s=1}^{\infty} \frac{sq^s}{1 - q^s} - \left(\sum_{s=1}^{\infty} \frac{sq^s}{1 - q^s} - \sum_{s=1}^{\infty} \frac{(2s-1)q^{2s-1}}{1 - q^{2s-1}} \right) \\
&= \sum_{s=1}^{\infty} \frac{(2s-1)q^{2s-1}}{1 - q^{2s-1}}.
\end{aligned}$$

Therefore, using equations (5)–(7), we have

$$\sum_{m=1}^{\infty} \left(\sum_{\lambda \in A_3(m)} l(\lambda) - \sum_{\lambda \in A_4(m)} l(\lambda) \right) q^m = \prod_{k=1}^{\infty} (1 + q^k)^k \sum_{s=1}^{\infty} \frac{sq^{2s}}{1 - q^{2s}}. \quad (8)$$

Combining (3), (4), and (8) gives the result.

To prove an n -color analogue of Beck's second conjecture, let us define $A_5(m)$ to be the set of n -color partitions of m such that there is exactly one part occurring thrice while all the other parts occur only once.

Theorem 2.2. *For all $m \geq 1$,*

$$|A_5(m)| = \sum_{\lambda \in A_4(m)} l(\lambda) - \sum_{\lambda \in A_3(m)} \bar{l}(\lambda).$$

Proof. For the partitions in the set $A_5(m)$ exactly one part occurs three times, while all the other parts occur only once. Let us assume that the part s_p occurs three times. Then it contributes a factor $sq^{3s}(1+q^s)^{s-1}$. Thus,

$$\begin{aligned} \sum_{m=1}^{\infty} |A_5(m)| q^m &= \sum_{s=1}^{\infty} sq^{3s}(1+q^s)^{s-1} \prod_{\substack{k=1 \\ k \neq s}}^{\infty} (1+q^k)^k \\ &= \prod_{k=1}^{\infty} (1+q^k)^k \sum_{s=1}^{\infty} \frac{sq^{3s}}{1+q^s}. \end{aligned} \quad (9)$$

Also,

$$\begin{aligned} \sum_{m=1}^{\infty} \left(\sum_{\lambda \in A_3(m)} \bar{l}(\lambda) \right) q^m &= \frac{\partial}{\partial x} \Big|_{x=1} \left(\prod_{k=1}^{\infty} \left(1 + \frac{xq^{2k-1}}{1-q^{2k-1}} \right)^{2k-1} \left(1 + \frac{xq^{2k}}{1-q^{2k}} \right)^k \right) \\ &= \sum_{s=1}^{\infty} \frac{sq^{2s}}{1-q^{2s}} \prod_{k=1}^{\infty} \left(1 + \frac{q^{2k-1}}{1-q^{2k-1}} \right)^{2k-1} \left(1 + \frac{q^{2k}}{1-q^{2k}} \right)^{k-1} \\ &\quad + \sum_{s=1}^{\infty} \frac{(2s-1)q^{2s-1}}{1-q^{2s-1}} \prod_{k=1}^{\infty} \left(1 + \frac{q^{2k-1}}{1-q^{2k-1}} \right)^{2k-2} \left(1 + \frac{q^{2k}}{1-q^{2k}} \right)^k \\ &= \prod_{k=1}^{\infty} \frac{1}{(1-q^{2k})^k (1-q^{2k-1})^{2k-1}} \left(\sum_{s=1}^{\infty} sq^{2s} + \sum_{s=1}^{\infty} (2s-1)q^{2s-1} \right) \\ &= \prod_{k=1}^{\infty} (1+q^k)^k \left(\sum_{s=1}^{\infty} sq^{2s} + \sum_{s=1}^{\infty} sq^s - \sum_{s=1}^{\infty} 2sq^{2s} \right) \quad (\text{by (1)}) \\ &= \prod_{k=1}^{\infty} (1+q^k)^k \sum_{s=1}^{\infty} sq^s (1-q^s) \\ &= \prod_{k=1}^{\infty} (1+q^k)^k \left(\sum_{s=1}^{\infty} \frac{sq^s}{1+q^s} - \sum_{s=1}^{\infty} \frac{sq^{3s}}{1+q^s} \right). \end{aligned} \quad (10)$$

Thus, using equations (6) and (10), we get

$$\sum_{m=1}^{\infty} \left(\sum_{\lambda \in A_4(m)} l(\lambda) - \sum_{\lambda \in A_3(m)} \bar{l}(\lambda) \right) q^m = \prod_{k=1}^{\infty} (1 + q^k)^k \sum_{s=1}^{\infty} \frac{sq^{3s}}{1 + q^s}.$$

The proof is complete.

Next, we prove a Beck type identity for the Euler type identity given by (2). To obtain this, we define the following sets of n -color partitions:

$A_6(m)$ – The set of n -color partitions of a positive integer m with even and distinct parts.

$A_7(m)$ – The set of n -color partitions of a positive integer m with even parts where parts divisible by 4 occur with odd subscripts only.

Theorem 2.3. *For all $m \geq 1$, let $A_8(m)$ be the set of n -color partitions of m into even parts where exactly one part occurs three times while all the other parts are distinct.*

$$|A_8(m)| = \sum_{\lambda \in A_6(m)} l(\lambda) - \sum_{\lambda \in A_7(m)} \bar{l}(\lambda).$$

Proof. We have the following generating function for $|A_8(m)|$:

$$\begin{aligned} \sum_{m=1}^{\infty} |A_8(m)| q^m &= \sum_{s=1}^{\infty} 2sq^{6s} (1 + q^{2s})^{2s-1} \prod_{\substack{k=1 \\ k \neq s}}^{\infty} (1 + q^{2k})^{2k} \\ &= \prod_{k=1}^{\infty} (1 + q^{2k})^{2k} \sum_{s=1}^{\infty} \frac{2sq^{6s}}{1 + q^{2s}}. \end{aligned}$$

Now,

$$\begin{aligned} \sum_{m=1}^{\infty} \left(\sum_{\lambda \in A_6(m)} l(\lambda) \right) q^m &= \frac{\partial}{\partial x} \Big|_{x=1} \left(\prod_{k=1}^{\infty} (1 + xq^{2k})^{2k} \right) \\ &= \sum_{s=1}^{\infty} 2sq^{2s} (1 + q^{2s})^{2s-1} \prod_{\substack{k=1 \\ k \neq s}}^{\infty} (1 + q^{2k})^{2k} \\ &= \prod_{k=1}^{\infty} (1 + q^{2k})^{2k} \sum_{s=1}^{\infty} \frac{2sq^{2s}}{1 + q^{2s}}. \end{aligned}$$

Also,

$$\begin{aligned}
& \sum_{m=1}^{\infty} \left(\sum_{\lambda \in A_7(m)} \bar{l}(\lambda) \right) q^m \\
&= \frac{\partial}{\partial x} \Big|_{x=1} \prod_{k=1}^{\infty} \left(1 + \frac{xq^{4k-2}}{1-q^{4k-2}} \right)^{4k-2} \left(1 + \frac{xq^{4k}}{1-q^{4k}} \right)^{2k} \\
&= \prod_{k=1}^{\infty} \frac{1}{(1-q^{4k})^{2k}(1-q^{4k-2})^{4k-2}} \left(\sum_{s=1}^{\infty} 2sq^{4s} + \sum_{s=1}^{\infty} (4s-2)q^{4s-2} \right) \\
&= \prod_{k=1}^{\infty} \frac{1}{(1-q^{4k})^{2k}(1-q^{4k-2})^{4k-2}} \left(\sum_{s=1}^{\infty} 2sq^{4s} + \sum_{s=1}^{\infty} 2sq^{2s} - \sum_{s=1}^{\infty} 4sq^{4s} \right) \\
&= \prod_{k=1}^{\infty} \frac{1}{(1-q^{4k})^{2k}(1-q^{4k-2})^{4k-2}} \left(\sum_{s=1}^{\infty} 2sq^{2s}(1-q^{2s}) \right) \\
&= \prod_{k=1}^{\infty} \frac{1}{(1-q^{4k})^{2k}(1-q^{4k-2})^{4k-2}} \left(\sum_{s=1}^{\infty} \frac{2sq^{2s}}{1+q^{2s}} - \sum_{s=1}^{\infty} \frac{2sq^{6s}}{1+q^{2s}} \right) \\
&= \prod_{k=1}^{\infty} (1+q^{2k})^{2k} \left(\sum_{s=1}^{\infty} \frac{2sq^{2s}}{1+q^{2s}} - \sum_{s=1}^{\infty} \frac{2sq^{6s}}{1+q^{2s}} \right). \quad (using (2))
\end{aligned}$$

Hence, we have

$$\sum_{m=1}^{\infty} \left(\sum_{\lambda \in A_6(m)} l(\lambda) - \sum_{\lambda \in A_7(m)} \bar{l}(\lambda) \right) q^m = \prod_{k=1}^{\infty} (1+q^{2k})^{2k} \sum_{s=1}^{\infty} \frac{2sq^{6s}}{1+q^{2s}}.$$

3. Dual Form of Beck Type Identities for n -Color Partitions

In this section, we prove a Beck type identity in the dual form. This identity involves four new sets of restricted partitions, which we define as follows:

$A_9(m)$ – The set of n -color partitions of a positive integer m having exactly one even part with even subscript and it occurs an even number of times while all the other parts are either even with odd subscripts or odd with any subscript.

$A_{10}(m)$ – The set of n -color partitions of a positive integer m having exactly one even part with even subscript and it occurs an odd number of times while all the other parts are either even with odd subscripts or parts are odd with any subscript.

$A_{11}(m)$ – The set of n -color partitions of a positive integer m such that there is exactly one repeated part and this part is even with even subscript while all the

other parts are distinct.

$A_{12}(m)$ – The set of n -color partitions of a positive integer m in which exactly one part (except even part with an even subscript) occurs more than once and all the other parts are distinct.

Theorem 3.1. For $m \geq 1$,

$$|A_9(m)| - |A_{10}(m)| = \sum_{\lambda \in A_4(m)} l_e(\lambda) = |A_{12}(m)| - |A_{11}(m)|.$$

Proof. Using an argument similar to the previous theorems, we can write the generating functions for $|A_9(m)|$ and $|A_{10}(m)|$.

$$\begin{aligned} \sum_{m=1}^{\infty} |A_9(m)| q^m &= \prod_{k=1}^{\infty} \frac{1}{(1 - q^{2k})^k (1 - q^{2k-1})^{2k-1}} \sum_{s=1}^{\infty} \frac{sq^{4s}}{1 - q^{4s}}, \\ \sum_{m=1}^{\infty} |A_{10}(m)| q^m &= \prod_{k=1}^{\infty} \frac{1}{(1 - q^{2k})^k (1 - q^{2k-1})^{2k-1}} \sum_{s=1}^{\infty} \frac{sq^{2s}}{1 - q^{4s}}. \end{aligned}$$

Consequently, we get

$$\begin{aligned} \sum_{m=1}^{\infty} (|A_{10}(m)| - |A_9(m)|) q^m &= \left(\sum_{s=1}^{\infty} \frac{sq^{2s}}{1 - q^{4s}} - \sum_{s=1}^{\infty} \frac{sq^{4s}}{1 - q^{4s}} \right) \\ &\quad \times \prod_{k=1}^{\infty} \frac{1}{(1 - q^{2k})^k (1 - q^{2k-1})^{2k-1}} \\ &= \prod_{k=1}^{\infty} (1 + q^k)^k \sum_{s=1}^{\infty} \frac{sq^{2s}}{1 + q^{2s}}. \end{aligned} \tag{11}$$

Now, considering the generating function for n -color partitions into distinct parts and introducing a new variable x to keep track of the number of even parts with even subscripts, we have

$$\sum_{m=1}^{\infty} \left(\sum_{\lambda \in A_4(m)} l_e(\lambda) \right) q^m = \prod_{k=1}^{\infty} (1 + q^{2k-1})^{2k-1} (1 + q^{2k})^k \frac{\partial}{\partial x} \Big|_{x=1} \prod_{s=1}^{\infty} (1 + xq^{2s})^s$$

$$\begin{aligned}
&= \prod_{k=1}^{\infty} (1 + q^{2k-1})^{2k-1} (1 + q^{2k})^k \prod_{\substack{k=1 \\ k \neq s}}^{\infty} (1 + q^{2k})^k \sum_{s=1}^{\infty} s q^{2s} (1 + q^{2s})^{s-1} \\
&= \prod_{k=1}^{\infty} (1 + q^k)^k \sum_{s=1}^{\infty} \frac{s q^{2s}}{1 + q^{2s}}.
\end{aligned} \tag{12}$$

Now, we write the generating functions for $|A_{11}(m)|$ and $|A_{12}(m)|$.

$$\begin{aligned}
\sum_{m=1}^{\infty} |A_{11}(m)| q^m &= \prod_{k=1}^{\infty} (1 + q^k)^k \sum_{s=1}^{\infty} \frac{s q^{4s}}{1 - q^{4s}}, \\
\sum_{m=1}^{\infty} |A_{12}(m)| q^m &= \prod_{k=1}^{\infty} (1 + q^k)^k \left(\sum_{s=1}^{\infty} \frac{s q^{4s}}{1 - q^{4s}} + \sum_{\substack{s=1 \\ s \text{ is odd}}}^{\infty} \frac{s q^{2s}}{1 - q^{2s}} \right).
\end{aligned}$$

Thus,

$$\begin{aligned}
\sum_{m=1}^{\infty} (|A_{12}(m)| - |A_{11}(m)|) q^m &= \prod_{k=1}^{\infty} (1 + q^k)^k \sum_{\substack{s=1 \\ s \text{ is odd}}}^{\infty} \frac{s q^{2s}}{1 - q^{2s}} \\
&= \prod_{k=1}^{\infty} (1 + q^k)^k \left(\sum_{s=1}^{\infty} \frac{s q^{2s}}{1 - q^{2s}} - \sum_{s=1}^{\infty} \frac{2s q^{4s}}{1 - q^{4s}} \right) \\
&= \prod_{k=1}^{\infty} (1 + q^k)^k \sum_{s=1}^{\infty} \frac{s q^{2s}}{1 + q^{2s}}.
\end{aligned} \tag{13}$$

Combining (11), (12) and (13) yields the desired result.

References

- [1] Agarwal, A. K., n -Color analogues of Gaussian polynomials, *Ars Combin.*, 61 (2001), 97-117.
- [2] Agarwal, A. K., Andrews, G. E., Rogers-Ramanujan identities for partitions with " n copies of n ", *J. Combin. Theory Ser. A*, 45 (1987), 40-49.
- [3] Andrews, G. E., Euler's partition identity and two problems of George Beck, *Math. Student*, 86 (2017), 115-119.
- [4] Ballantine, C., Bielak, R., Combinatorial proofs of two Euler-type identities due to Andrews, *Ann. Comb.*, 23 (2019), 511-525.

- [5] Chern, S., On a conjecture of George Beck, *Int. J. Number Theory*, 14 (2018), 647-651.
- [6] Fu, S., Tang, D., Generalizing a partition theorem of Andrews, *Math. Student*, 86 (2017), 91–96.
- [7] Li, R., Wang, A. Y. Z., The dual form of Beck type identities, *Ramanujan J.*, 58(1) (2021), 145-165.
- [8] Li, R., Wang, A. Y. Z., Composition analogues of Beck’s conjectures on partitions, *European J. Combin.*, 81 (2019), 210-220.
- [9] Li, R., Wang, A. Y. Z., Generalization of two problems of George Beck, *Discrete Math.*, 343(5) (2020), 111805, 12.
- [10] Sloane, N. J. A., The On-Line Encyclopedia of Integer Sequences, Published electronically at <https://oeis.org>.
- [11] Yang, J. Y. X., Combinatorial proofs and generalizations of conjectures related to Euler’s partition theorem, *European J. Combin.*, 76 (2019), 62-72.

This page intentionally left blank.