

MICRO αg -CONTINUOUS MAPS IN MICRO TOPOLOGICAL SPACES

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Abstract: In this paper, we introduce and study the properties of Micro αg -closure operator and Micro αg -interior operator. As an application of Micro αg -closed sets, we introduce four new spaces namely, Micro α -space, Micro ${}_{\alpha}T_b$ -space, Micro ${}_{1/2}T_{\alpha}$ -space, Micro ${}_{\alpha}T_d$ -space and analyze their properties and interrelations. Further, we introduce a new notion of Micro continuous maps called Micro αg -continuous maps with the help of Micro αg -closed sets. Also, we obtain interrelationships between the newly defined maps and already existing various Micro continuous maps. Finally, we derive some basic properties of Micro αg -continuous maps via separation axioms.

Keywords and Phrases: Micro αg -closed set, Micro αg -interior, Micro αg -closure, separation axioms, Micro continuous maps, Micro αg -continuous maps.

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1. Introduction

Topology provides a general framework for studying spatial structures and properties that remain invariant under suitable mappings. In recent years, generalized topological structures have received considerable attention because they provide flexible tools for studying sets and mappings when the usual open and closed set conditions are too restrictive. In particular, generalized open sets, generalized

closed sets and generalized forms of continuity have been extensively investigated in both classical and approximation-based topological settings.

Nano topology was introduced by Carmel Richard in 2013 [3]. It was introduced as a finite approximation-based topological structure and has subsequently motivated several developments in generalized topology. Micro topology extends this framework by providing a more flexible approximation structure for studying subsets of a universe. The use of Micro topological spaces is therefore important in investigating generalized topological properties through approximation-based information. In this setting, notions such as Micro open sets [10], Micro closed sets [10], Micro interior [10] and Micro closure [10] provide the basic tools for studying structural properties and mappings. Since generalized topological properties are often studied through their behaviour under mappings, the investigation of generalized continuity in Micro topological spaces also contributes to the study of topological invariance and preservation phenomena.

Several generalized closed sets have been introduced in Micro topological spaces. Among them are Micro α -closed sets [4, 9], Micro g -closed sets [5, 11], Micro β -closed sets [6] and Micro b -closed sets [8], each of which is associated with a particular generalized openness condition. These classes enlarge the class of Micro closed sets and provide useful methods for studying different forms of non-closedness. However, each individual generalized structure captures only a particular aspect of the behaviour of a set. Consequently, a set that does not satisfy the specific condition required for Micro α -closedness [4] or Micro g -closedness [5] may still possess a meaningful generalized closed behaviour that cannot be adequately described by either structure alone. Similar limitations arise when different generalized closed classes are considered separately.

This motivates the study of Micro αg -closed sets. The purpose of introducing Micro αg -closed sets [1] is not merely to add another generalized closed set class, but to provide a more flexible framework in which the characteristics associated with the Micro α and Micro g -structures can be considered together. The resulting class contains important previously studied classes as subclasses. In particular, the relationships $Mic\text{-}closed \subseteq Mic\text{-}\alpha\text{-}closed \subseteq Mic\text{-}\alpha g\text{-}closed$ and $Mic\text{-}closed \subseteq Mic\text{-}g\text{-}closed \subseteq Mic\text{-}\alpha g\text{-}closed$ demonstrate the more inclusive nature of the Micro αg -closed structure. Thus, Micro αg -closed sets provide an additional level of generalized closedness and allow sets to be studied beyond the restrictions imposed by the individual Micro α and Micro g -structures.

Associated with a generalized class of closed sets, closure and interior operators are fundamental tools for understanding the structure of subsets. The Micro αg -closure of a set describes the smallest Micro αg -closed set containing it and therefore

provides information about its generalized boundary behaviour. In contrast, the Micro αg -interior identifies the largest Micro αg -open set contained in the given set and describes the portion of the set satisfying the generalized αg -openness condition. These operators are useful not only for characterizing Micro αg -closed and Micro αg -open sets, but also for comparing the new structure with existing Micro closure and interior operators. In particular, the Micro αg -closure need not behave in exactly the same way as the Micro α -closure or Micro g -closure, and examples can therefore reveal generalized boundary behavior that is not visible through the existing operators.

Generalized separation properties [2, 7] provide another important direction in Micro topology. Various Micro separation spaces, including Micro α -space, Micro ${}_{\alpha}T_b$ -space, Micro ${}_{1/2}T_{\alpha}$ -space and Micro ${}_{\alpha}T_d$ -spaces, describe different relationships between generalized closed sets and Micro closed sets. Studying these spaces in relation to the Micro αg -structure helps to identify conditions under which the generalized structure coincides with, or differs from, previously established Micro topological structures. Such comparisons are useful for understanding the structural position of Micro αg -closed sets within the hierarchy of generalized Micro topological spaces.

Continuity is one of the fundamental notions through which topological structures and their invariance are studied. Several generalized forms of continuity have been introduced by replacing open or closed sets with suitable generalized classes. Motivated by the Micro αg -closed structure, Micro αg -continuity is considered as a natural extension of generalized continuity in Micro topological spaces. A Micro αg -continuous map requires the inverse image of a Micro closed set to be Micro αg -closed. Since Micro αg -closed sets form a more inclusive class than Micro closed, Micro α -closed and Micro g -closed sets, this notion provides a broader framework for studying mappings that may not satisfy the corresponding stronger continuity conditions. The study of composition also reveals that additional structural assumptions, such as suitable separation properties of the intermediate Micro topological space.

Although generalized closed sets and generalized continuous maps have been studied extensively in Micro topological spaces, the relationships among Micro αg -closed sets, their closure and interior operators, separation properties and generalized continuity have not been sufficiently developed as a unified framework. In particular, the interaction between Micro αg -closure and existing Micro closure operators, the structural role of the associated separation spaces, and the preservation and composition properties of Micro αg -continuous mappings require further investigation.

The main objectives of this paper are therefore to investigate the Micro αg -closure and Micro αg -interior operators and establish their fundamental properties, to compare these operators with the existing Micro closure and interior structures, to study the relationships among Micro α , Micro ${}_{\alpha}T_b$, Micro ${}_{1/2}T_{\alpha}$ and Micro ${}_{\alpha}T_d$ -spaces; and to develop the theory of Micro αg -continuous maps through characterizations, examples and composition results. Particular attention is given to examples and counterexamples that demonstrate the distinct behaviour and conceptual significance of the proposed Micro αg -structure rather than merely verifying the definitions.

The paper is organized as follows. Section 2 recalls the basic concepts required for Micro topological spaces and the generalized closed sets used in the sequel. Section 3 deals with Micro αg -closure and Micro αg -interior and discusses their properties and relationships with existing operators. Section 4 investigates the relevant Micro separation spaces and their structural relationships. Section 5 is devoted to Micro αg -continuous maps and their characterizations and composition properties. Finally, the main findings and possible directions for further investigation are summarized in the conclusion.

2. Preliminaries

Throughout this paper $(\Delta, \tau_R(X^*), \mu_R(X^*))$, $(\Omega, \tau_R(Y^*), \mu_R(Y^*))$ and $(\psi, \tau_R(Z^*), \mu_R(Z^*))$ (or simply Δ, Ω and ψ) denote Micro topological spaces on which no separation axioms are assumed unless explicitly stated.

Definition 2.1. [3] *Let Δ be the universe, R be an equivalence relation on Δ and $\tau_R(X^*) = \{\Delta, \phi, L_R(X^*), U_R(X^*), B_R(X^*)\}$, where $X^* \subseteq \Delta$. Then $\tau_R(X^*)$ satisfies the following axioms:*

1. $\Delta, \phi \in \tau_R(X^*)$.
2. *The union of the elements of any sub-collection of $\tau_R(X^*)$ is in $\tau_R(X^*)$.*
3. *The intersection of the elements of any finite sub-collection of $\tau_R(X^*)$ is in $\tau_R(X^*)$.*

That is, $\tau_R(X^*)$ is a topology on Δ called the Nano topology on Δ with respect to X^* . We call $(\Delta, \tau_R(X^*))$ as the Nano topological space. The elements of $\tau_R(X^*)$ are called as Nano open sets and the complement of a Nano open set is called a Nano closed set.

Definition 2.2. [10] *Let $(\Delta, \tau_R(X^*))$ be a Nano topological space. Then $\mu_R(X^*) = \{N \cup (N' \cap \mu) : N, N' \in \tau_R(X^*)\}$ and $\mu \notin \tau_R(X^*)$ and $\mu_R(X^*)$ satisfies the following axioms:*

1. $\Delta, \phi \in \mu_R(X^*)$.
2. The union of the elements of any sub-collection of $\mu_R(X^*)$ is in $\mu_R(X^*)$.
3. The intersection of the elements of any finite sub-collection of $\mu_R(X^*)$ is in $\mu_R(X^*)$.

That is $\mu_R(X^*)$ is a topology on Δ called the Micro topology on Δ with respect to X^* . The triplet $(\Delta, \tau_R(X^*), \mu_R(X^*))$ is called Micro topological space and the elements of $\mu_R(X^*)$ are called Micro open (briefly Mic-op.) sets and the complement of a Mic-op. set is called a Micro closed (briefly Mic-cl.) set.

Definition 2.3. [10] Let $(\Delta, \tau_R(X^*), \mu_R(X^*))$ be a Micro topological space and $D \subseteq \Delta$. Then the Micro closure of a set D is denoted by $\text{Mic-cl}(D)$ and is defined as $\text{Mic-cl}(D) = \bigcap \{F : F \text{ is Mic-cl. in } \Delta \text{ and } D \subseteq F\}$.

The Micro interior of a set D is denoted by $\text{Mic-int}(D)$ and is defined as $\text{Mic-int}(D) = \bigcup \{F : F \text{ is Mic-op. in } \Delta \text{ and } D \supseteq F\}$.

Definition 2.4. Let $(\Delta, \tau_R(X^*), \mu_R(X^*))$ be a Micro topological space and $D \subseteq \Delta$. Then D is said to be a

1. Micro pre-open (briefly $\text{Mic}_P\text{-op.}$) set if $D \subseteq \text{Mic-int}(\text{Mic-cl}(D))$. [10]
2. Micro semi-open (briefly $\text{Mic}_S\text{-op.}$) set if $D \subseteq \text{Mic-cl}(\text{Mic-int}(D))$. [10]
3. Micro α -open (briefly $\text{Mic}_\alpha\text{-op.}$) set if $D \subseteq \text{Mic-int}(\text{Mic-cl}(\text{Mic-int}(D)))$. [4]
4. Micro β -open (briefly $\text{Mic}_\beta\text{-op.}$) set if $D \subseteq \text{Mic-cl}(\text{Mic-int}(\text{Mic-cl}(D)))$. [6]
5. Micro generalized closed (briefly $\text{Mic}_g\text{-cl.}$) if $\text{Mic-cl}(D) \subseteq L$ whenever $D \subseteq L$ and L is Mic-op. in Δ . [5]

The complements of the above mentioned sets are called their respective Mic-cl. and Mic-op. sets.

Definition 2.5. [4] Let $(\Delta, \tau_R(X^*), \mu_R(X^*))$ be a Micro topological space and $D \subseteq \Delta$. Then the Micro α -closure of a set D is denoted by $\text{Mic-}\alpha\text{cl}(D)$ and is defined as

$$\text{Mic-}\alpha\text{cl}(D) = \bigcap \{F : F \text{ is Mic-}\alpha\text{-cl. in } \Delta \text{ and } D \subseteq F\}$$

The Micro α -interior of a set D is denoted by $\text{Mic-}\alpha\text{int}(D)$ and is defined as $\text{Mic-}\alpha\text{int}(D) = \bigcup \{F : F \text{ is Mic-}\alpha\text{-op. in } \Delta \text{ and } D \supseteq F\}$.

Definition 2.6. Let $(\Delta, \tau_R(X^*), \mu_R(X^*))$ be a Micro topological space and $D \subseteq \Delta$.

Then the Micro g -closure of a set D is denoted by $\text{Mic-gcl}(D)$ and is defined as $\text{Mic-gcl}(D) = \bigcap \{F : F \text{ is Mic}_g\text{-cl. in } \Delta \text{ and } D \subseteq F\}$

The Micro g -interior of a set D is denoted by $\text{Mic-gint}(D)$ and is defined as $\text{Mic-gint}(D) = \bigcup \{F : F \text{ is Mic}_g\text{-op. in } \Delta \text{ and } D \supseteq F\}$.

Definition 2.7. [5] A Micro topological space $(\Delta, \tau_R(X^*), \mu_R(X^*))$ is said to be a Micro $T_{1/2}$ -space, if every $\text{Mic}_g\text{-cl.}$ set in Δ is Mic-closed.

Definition 2.8. [1] Let $(\Delta, \tau_R(X^*), \mu_R(X^*))$ be a Micro topological space. A subset D of Δ is said to be Micro αg -closed (briefly $\text{Mic}_{\alpha g}\text{-cl.}$), if $\text{Mic-}\alpha\text{cl}(D) \subseteq L$ whenever $D \subseteq L$ and L is Mic-op. in Δ .

Lemma 2.1. [1] Let D be $\text{Mic}_{\alpha g}\text{-cl.}$ in Δ . Then $\text{Mic-}\alpha\text{cl}(D) - D$ contains no non empty Mic-cl. set.

Lemma 2.2. [1] Let $(\Delta, \tau_R(X^*), \mu_R(X^*))$ be a Micro topological space. Then, for each $x \in \Delta$, either $\{x\}$ is a Mic-cl. set or $\Delta - \{x\}$ is a $\text{Mic}_{\alpha g}\text{-cl.}$ set.

Result 2.1.

1. Every Mic-cl. set is Micro α -closed (briefly $\text{Mic}_\alpha\text{-cl.}$). [4]
2. Every Mic-cl. set is Micro g -closed (briefly $\text{Mic}_g\text{-cl.}$). [5]
3. Every Mic-cl. set is $\text{Mic}_{\alpha g}\text{-closed}$. [1]
4. Every $\text{Mic}_\alpha\text{-cl.}$ set is $\text{Mic}_{\alpha g}\text{-closed}$. [1]
5. Every $\text{Mic}_g\text{-cl.}$ set is $\text{Mic}_{\alpha g}\text{-closed}$. [1]

Note that $\text{Mic}_{\alpha g}\text{-cl.}$ sets are dependent with existing Mic-cl. , $\text{Mic}_\alpha\text{-cl.}$, $\text{Mic}_g\text{-cl.}$ sets and independent with $\text{Mic}_P\text{-cl.}$, $\text{Mic}_S\text{-cl.}$, $\text{Mic}_\beta\text{-cl.}$ sets.

Theorem 2.3. [2] Let $D \subseteq \Delta$. Then $x \in \text{Mic-}\alpha g\text{-cl}(D)$ iff for every Micro αg -open (briefly $\text{Mic}_{\alpha g}\text{-op.}$) subset L of Δ containing x , $L \cap D \neq \emptyset$.

Definition 2.9. A map $v : (\Delta, \tau_R(X^*), \mu_R(X^*)) \rightarrow (\Omega, \tau_R(Y^*), \mu_R(Y^*))$ is called

1. Micro continuous (briefly Mic-cont.) [10] if $v^{-1}(P)$ is Mic-cl. in Δ for every Mic-cl. set P in Ω .
2. Micro pre-continuous (briefly $\text{Mic}_P\text{-cont.}$) [10] if $v^{-1}(P)$ is Micro pre-closed (briefly $\text{Mic}_P\text{-cl.}$) in Δ for every Mic-cl. set P in Ω .
3. Micro semi-continuous (briefly $\text{Mic}_S\text{-cont.}$) [10] if $v^{-1}(P)$ is Micro semi-closed (briefly $\text{Mic}_S\text{-cl.}$) in Δ for every Mic-cl. set P in Ω .

4. Micro α -continuous (briefly $\text{Mic}_\alpha\text{-cont.}$) [4] if $v^{-1}(P)$ is $\text{Mic}_\alpha\text{-cl.}$ in Δ for every Mic-cl. set P in Ω .
5. Micro β -continuous (briefly $\text{Mic}_\beta\text{-cont.}$) [6] if $v^{-1}(P)$ is Micro β -closed (briefly $\text{Mic}_\beta\text{-cl.}$) in Δ for every Mic-cl. set P in Ω .
6. Micro g -continuous (briefly $\text{Mic}_g\text{-cont.}$) [11] if $v^{-1}(P)$ is $\text{Mic}_g\text{-cl.}$ in Δ for every Mic-cl. set P in Ω .

3. Micro αg -closure operator and Micro αg -interior operator

The Micro closure of a subset D is the smallest Mic-cl. set containing D . By contrast, $\text{Mic-}\alpha g\text{-cl}(D)$ is formed using the larger class of $\text{Mic}_{\alpha g}\text{-cl.}$ supersets.

Definition 3.1. Let $(\Delta, \tau_R(X^*), \mu_R(X^*))$ be a Micro topological space and $D \subseteq \Delta$. Then the Micro αg -closure of a set D is denoted by $\text{Mic-}\alpha g\text{-cl}(D)$ and is defined as

$$\text{Mic-}\alpha g\text{-cl}(D) = \cap \{T : T \text{ is } \text{Mic}_{\alpha g}\text{-cl. in } \Delta \text{ and } D \subseteq T\}.$$

The $\text{Mic-}\alpha g\text{-cl}(D)$ may be coarser than $\text{Mic-cl}(D)$.

Remark 3.1. For a subset D of Δ ,

1. $D \subseteq \text{Mic-}\alpha g\text{-cl}(D) \subseteq \text{Mic-cl}(D)$
2. $D \subseteq \text{Mic-}\alpha g\text{-cl}(D) \subseteq \text{Mic-}\alpha\text{cl}(D)$
3. $D \subseteq \text{Mic-}\alpha g\text{-cl}(D) \subseteq \text{Mic-gcl}(D)$

(1) The first inclusion is trivially true. The second inclusion is a consequence of the fact that every Mic-cl. set is $\text{Mic}_{\alpha g}\text{-closed}$.

(2) The first inclusion is trivially true. The second inclusion is a consequence of the fact that every $\text{Mic}_\alpha\text{-cl.}$ set is $\text{Mic}_{\alpha g}\text{-closed}$.

(3) The first inclusion is trivially true. The second inclusion is a consequence of the fact that every $\text{Mic}_g\text{-cl.}$ set is $\text{Mic}_{\alpha g}\text{-closed}$.

Hence, the Micro αg -closure is coarser than the Micro closure, Micro α -closure and Micro g -closure.

Example 3.1. Let $\Delta = \{e, u, g\}$, $\Delta/R = \{\{e\}, \{u, g\}\}$. Let $X^* = \{e\} \subseteq \Delta$. Then $\tau_R(X^*) = \{\Delta, \phi, \{e\}\}$. Let $\mu = \{e, u\} \notin \tau_R(X^*)$. Then $\mu_R(X^*) = \{\phi, \{e\}, \{e, u\}, \Delta\}$. Let $D = \{u\}$. Then $\text{Mic-}\alpha g\text{-cl}(D) = \{u\}$ and $\text{Mic-cl}(D) = \{u, g\}$ and so $D \subseteq \text{Mic-}\alpha g\text{-cl}(D) \subseteq \text{Mic-cl}(D)$.

Example 3.2. Let $\Delta = \{k, f, d\}$, $\Delta/R = \{\{k\}, \{f, d\}\}$. Let $X^* = \{f\} \subseteq \Delta$. Then $\tau_R(X^*) = \{\Delta, \phi\}$. Let $\mu = \{f\} \notin \tau_R(X^*)$. Then $\mu_R(X^*) = \{\phi, \{f\}, \Delta\}$. Let

$D = \{f\}$. Then $\text{Mic-}\alpha g\text{-cl}(D) = \{f\}$ and $\text{Mic-}\alpha cl(D) = \Delta$ and so $D \subseteq \text{Mic-}\alpha g\text{-cl}(D) \subseteq \text{Mic-}\alpha cl(D)$.

Example 3.3. Let $\Delta = \{h, t, r\}$, $\Delta/R = \{\{h\}, \{t, r\}\}$. Let $X^* = \{h\} \subseteq \Delta$. Then $\tau_R(X^*) = \{\Delta, \phi, \{h\}\}$. Let $\mu = \{h, t\} \notin \tau_R(X^*)$. Then $\mu_R(X^*) = \{\phi, \{h\}, \{h, t\}, \Delta\}$. Let $D = \{t\}$. Then $\text{Mic-}\alpha g\text{-cl}(D) = \{t\}$ and $\text{Mic-}gcl(D) = \{t, r\}$ and so $D \subseteq \text{Mic-}\alpha g\text{-cl}(D) \subseteq \text{Mic-}gcl(D)$.

Remark 3.2. The inclusion relations given in Remark 3.1 (1) may be proper.

Example 3.4. Let $\Delta = \{e, u, g\}$, $\Delta/R = \{\{e\}, \{u, g\}\}$. Let $X^* = \{e\} \subseteq \Delta$. Then $\tau_R(X^*) = \{\Delta, \phi, \{e\}\}$. Let $\mu = \{e, u\} \notin \tau_R(X^*)$. Then $\mu_R(X^*) = \{\phi, \{e\}, \{e, u\}, \Delta\}$. Let $D = \{e\}$. Then $\text{Mic-}\alpha g\text{-cl}(D) = \{e, g\}$ and $\text{Mic-cl}(D) = \Delta$ and so $D \subset \text{Mic-}\alpha g\text{-cl}(D) \subset \text{Mic-cl}(D)$.

Proposition 3.1. Let D and T be any two subsets of Δ . Then the following statements are true.

- (a) $\text{Mic-}\alpha g\text{-cl}(\phi) = \phi$ and $\text{Mic-}\alpha g\text{-cl}(\Delta) = \Delta$.
- (b) $D \subseteq \text{Mic-}\alpha g\text{-cl}(D)$.
- (c) If T is any $\text{Mic}_{\alpha g}\text{-cl.}$ set containing D , then $\text{Mic-}\alpha g\text{-cl}(D) \subseteq T$.
- (d) If $D \subseteq T$, then $\text{Mic-}\alpha g\text{-cl}(D) \subseteq \text{Mic-}\alpha g\text{-cl}(T)$.
- (e) $\text{Mic-}\alpha g\text{-cl}(D) \cup \text{Mic-}\alpha g\text{-cl}(T) \subseteq \text{Mic-}\alpha g\text{-cl}(D \cup T)$.
- (f) $\text{Mic-}\alpha g\text{-cl}(D \cap T) \subseteq \text{Mic-}\alpha g\text{-cl}(D) \cap \text{Mic-}\alpha g\text{-cl}(T)$.
- (g) $\text{Mic-}\alpha g\text{-cl}(\text{Mic-}\alpha g\text{-cl}(D)) = \text{Mic-}\alpha g\text{-cl}(D)$.

Proof. (a) As $\text{Mic-}\alpha g\text{-cl}(D)$ is the intersection of all $\text{Mic}_{\alpha g}\text{-cl.}$ sets containing a subset D , $\text{Mic-}\alpha g\text{-cl}(\phi) = \phi$ and $\text{Mic-}\alpha g\text{-cl}(\Delta) = \Delta$.

(b) As $\text{Mic-}\alpha g\text{-cl}(D) = \cap \{T: T \text{ is } \text{Mic}_{\alpha g}\text{-cl. in } \Delta \text{ and } D \subseteq T\}$. Every set T in this family contains D . Therefore, their intersection also contains D . Hence, $D \subseteq \text{Mic-}\alpha g\text{-cl}(D)$.

(c) Let T be any $\text{Mic}_{\alpha g}\text{-cl.}$ set containing D . Since $\text{Mic-}\alpha g\text{-cl}(D)$ is the intersection of all $\text{Mic}_{\alpha g}\text{-cl.}$ sets containing D , $\text{Mic-}\alpha g\text{-cl}(D)$ is contained in every $\text{Mic}_{\alpha g}\text{-cl.}$ set containing D . Therefore $\text{Mic-}\alpha g\text{-cl}(D) \subseteq T$.

(d) As the definition of $\text{Mic-}\alpha g\text{-closure}$, $\text{Mic-}\alpha g\text{-cl}(D) \subseteq \text{Mic-}\alpha g\text{-cl}(T)$.

(e) Since $D \subseteq D \cup T$ and $T \subseteq D \cup T$ by (b) $\text{Mic-}\alpha g\text{-cl}(D) \subseteq \text{Mic-}\alpha g\text{-cl}(D \cup T)$ and $\text{Mic-}\alpha g\text{-cl}(T) \subseteq \text{Mic-}\alpha g\text{-cl}(D \cup T)$. Hence $\text{Mic-}\alpha g\text{-cl}(D) \cup \text{Mic-}\alpha g\text{-cl}(T) \subseteq \text{Mic-}\alpha g\text{-cl}(D \cup T)$.

(f) Since $D \cap T \subseteq D$ and $D \cap T \subseteq T$ by (b) $\text{Mic-}\alpha g\text{-cl}(D \cap T) \subseteq \text{Mic-}\alpha g\text{-cl}(D)$ and $\text{Mic-}\alpha g\text{-cl}(D \cap T) \subseteq \text{Mic-}\alpha g\text{-cl}(T)$. Therefore $\text{Mic-}\alpha g\text{-cl}(D \cap T) \subseteq \text{Mic-}\alpha g\text{-cl}(D) \cap \text{Mic-}\alpha g\text{-cl}(T)$.

(g) As the definition of $\text{Mic-}\alpha g\text{-closure}$, $\text{Mic-}\alpha g\text{-cl}(\text{Mic-}\alpha g\text{-cl}(D)) = \text{Mic-}\alpha g\text{-cl}(D)$.

Remark 3.3. Since $\text{Mic-}\alpha g\text{-cl}(D) \cup \text{Mic-}\alpha g\text{-cl}(T) \neq \text{Mic-}\alpha g\text{-cl}(D \cup T)$, Micro αg -closure is not a Kuratowski closure operator on $(\Delta, \tau_R(X^*), \mu_R(X^*))$.

Remark 3.4. In general, the reverse inclusion of (f) in Proposition 3.1 is not true.

Example 3.5. In Example 3.1, If $D = \{e\}$ and $T = \{g\}$ then $\text{Mic-}\alpha g\text{-cl}(D) = \{e, g\}$ and $\text{Mic-}\alpha g\text{-cl}(T) = \{g\}$, $D \cap T = \emptyset$, $\text{Mic-}\alpha g\text{-cl}(D \cap T) = \emptyset$. But $\text{Mic-}\alpha g\text{-cl}(D) \cap \text{Mic-}\alpha g\text{-cl}(T) = \{g\}$.

Proposition 3.2. Let D be any subset of Δ . If D is $\text{Mic}_{\alpha g}\text{-cl.}$ in Δ then $\text{Mic-}\alpha g\text{-cl}(D) = D$.

Proof. Let D be $\text{Mic}_{\alpha g}\text{-cl.}$ in Δ . By Definition 3.1, $D \subseteq \text{Mic-}\alpha g\text{-cl}(D)$. Also D is $\text{Mic}_{\alpha g}\text{-cl.}$ containing D , From Proposition 3.1 (c), $\text{Mic-}\alpha g\text{-cl}(D) \subseteq D$. Hence $\text{Mic-}\alpha g\text{-cl}(D) = D$.

Remark 3.5. Converse of Proposition 3.2 is not true.

Example 3.6. Let $\Delta = \{r, t, f, b\}$, $\Delta/R = \{\{r, t\}, \{f\}, \{b\}\}$. Let $X^* = \{f\} \subseteq \Delta$. Then $\tau_R(X^*) = \{\Delta, \emptyset, \{f\}\}$. Let $\mu = \{t\} \notin \tau_R(X^*)$. Then $\mu_R(X^*) = \{\emptyset, \{t\}, \{f\}, \{t, f\}, \Delta\}$. If $D = \{t, f\}$ then $\text{Mic-}\alpha g\text{-cl}(\{t, f\}) = \{t, f\}$, but D is not a $\text{Mic}_{\alpha g}\text{-cl.}$ set.

Remark 3.6. Let $D \subseteq \Delta$. Then $\text{Mic-}\alpha g\text{-cl}(D)$ need not be a $\text{Mic}_{\alpha g}\text{-cl.}$ set.

Example 3.7. Let $\Delta = \{f, e, t, c\}$, $\Delta/R = \{\{f, e\}, \{t\}, \{c\}\}$. Let $X^* = \{t\} \subseteq \Delta$. Then $\tau_R(X^*) = \{\Delta, \emptyset, \{t\}\}$. Let $\mu = \{e\} \notin \tau_R(X^*)$. Then $\mu_R(X^*) = \{\emptyset, \{e\}, \{t\}, \{e, t\}, \Delta\}$. Hence $\text{Mic-}\alpha g\text{-cl}(\{e\}) = \{e\} \neq \text{Mic}_{\alpha g}\text{-closed}$.

Definition 3.2. Let $(\Delta, \tau_R(X^*), \mu_R(X^*))$ be a Micro topological space and $D \subseteq \Delta$. Then the Micro αg -interior of a set D is denoted by $\text{Mic-}\alpha g\text{-int}(D)$ and is defined as $\text{Mic-}\alpha g\text{-int}(D) = \cup \{T : T \text{ is } \text{Mic}_{\alpha g}\text{-op. in } \Delta \text{ and } D \supseteq T\}$.

Remark 3.7. For a subset D of Δ ,

1. $\text{Mic-int}(D) \subseteq \text{Mic-}\alpha g\text{-int}(D) \subseteq D$
2. $\text{Mic-}\alpha \text{int}(D) \subseteq \text{Mic-}\alpha g\text{-int}(D) \subseteq D$
3. $\text{Mic-gint}(D) \subseteq \text{Mic-}\alpha g\text{-int}(D) \subseteq D$

Example 3.8. Let $\Delta = \{k, o, j\}$, $\Delta/R = \{\{k, o, j\}\}$. Let $X^* = \{k\} \subseteq \Delta$. Then $\tau_R(X^*) = \{\Delta, \emptyset\}$. Let $\mu = \{k\} \notin \tau_R(X^*)$. Then $\mu_R(X^*) = \{\emptyset, \{k\}, \Delta\}$. Let $D = \{k, j\}$. Then $\text{Mic-}\alpha g\text{-int}(D) = \{k, j\}$ and $\text{Mic-int}(D) = \{k\}$ and so $\text{Mic-int}(D) \subseteq \text{Mic-}\alpha g\text{-int}(D) \subseteq D$.

Example 3.9. In example 3.8, if $D = \{k, j\}$. Then $\text{Mic-}\alpha g\text{-int}(D) = \{k, j\}$ and $\text{Mic-}\alpha \text{int}(D) = \{k, j\}$ and so $\text{Mic-}\alpha \text{int}(D) \subseteq \text{Mic-}\alpha g\text{-int}(D) \subseteq D$.

Example 3.10. Let $\Delta = \{g, p, i\}, \Delta/R = \{\{g\}, \{p, i\}\}$. Let $X^* = \{p\} \subseteq \Delta$. Then $\tau_R(X^*) = \{\Delta, \phi, \{g\}\}$. Let $\mu = \{g, i\} \notin \tau_R(X^*)$. Then $\mu_R(X^*) = \{\phi, \{g\}, \{g, i\}, \Delta\}$. Let $D = \{p\}$. Then $\text{Mic-}\alpha g\text{-int}(D) = \{p\}$ and $\text{Mic-gint}(D) = \phi$ and so $\text{Mic-gint}(D) \subseteq \text{Mic-}\alpha g\text{-int}(D) \subseteq D$.

Proposition 3.3. Let D and T be any two subsets of $(\Delta, \tau_R(X^*), \mu_R(X^*))$. Then the following statements are true

- (a) $\text{Mic-}\alpha g\text{-int}(\phi) = \phi$ and $\text{Mic-}\alpha g\text{-int}(\Delta) = \Delta$.
- (b) $\text{Mic-}\alpha g\text{-int}(D) \subseteq D$.
- (c) If T is any $\text{Mic}_{\alpha g}$ -op. set contained in D , then $T \subseteq \text{Mic-}\alpha g\text{-int}(D)$.
- (d) If $D \subseteq T$ then $\text{Mic-}\alpha g\text{-int}(D) \subseteq \text{Mic-}\alpha g\text{-int}(T)$.
- (e) $\text{Mic-}\alpha g\text{-int}(D \cap T) \subseteq \text{Mic-}\alpha g\text{-int}(D) \cap \text{Mic-}\alpha g\text{-int}(T)$.
- (f) $\text{Mic-}\alpha g\text{-int}(D \cup T) \supseteq \text{Mic-}\alpha g\text{-int}(D) \cup \text{Mic-}\alpha g\text{-int}(T)$.
- (g) $\text{Mic-}\alpha g\text{-int}(\text{Mic-}\alpha g\text{-int}(D)) = \text{Mic-}\alpha g\text{-int}(D)$.

Proof: Similar to the Proposition 3.1.

Proposition 3.4. Let D be any subset of Δ . If D is $\text{Mic}_{\alpha g}$ -op. in $(\Delta, \tau_R(X^*), \mu_R(X^*))$ then $\text{Mic-}\alpha g\text{-int}(D) = D$.

Remark 3.8. Converse of Proposition 3.4 is not true.

Example 3.11. Let $\Delta = \{e, g, o, h\}, \Delta/R = \{\{e, g\}, \{o\}, \{h\}\}$. Let $X^* = \{o\} \subseteq \Delta$. Then $\tau_R(X^*) = \{\Delta, \phi, \{o\}\}$. Let $\mu = \{g\} \notin \tau_R(X^*)$. Then $\mu_R(X^*) = \{\phi, \{g\}, \{o\}, \{g, o\}, \Delta\}$. $\text{Mic-}\alpha g\text{-int}(\{e, h\}) = \{e, h\}$. But $\{e, h\}$ is not $\text{Mic}_{\alpha g}$ -op. in Δ .

4. Separation Axioms via Micro αg -closed sets

As an application of $\text{Mic}_{\alpha g}$ -cl. sets four new spaces namely, Micro α -space, Micro ${}_{\alpha}T_b$ -space, Micro ${}_{1/2}T_{\alpha}$ -space, Micro ${}_{\alpha}T_d$ -space are introduced and their properties and interrelations are studied.

Definition 4.1. A Micro topological space $(\Delta, \tau_R(X^*), \mu_R(X^*))$ is said to be a

1. Micro α -space, if every Mic_{α} -cl. set in Δ is Mic-closed.
2. Micro ${}_{\alpha}T_b$ -space, if every $\text{Mic}_{\alpha g}$ -cl. set in Δ is Mic-closed.
3. Micro ${}_{1/2}T_{\alpha}$ -space, if every $\text{Mic}_{\alpha g}$ -cl. set in Δ is Mic_{α} -closed.
4. Micro ${}_{\alpha}T_d$ -space, if every $\text{Mic}_{\alpha g}$ -cl. set in Δ is Mic_g -closed.

These four spaces describe different relationships between the $\text{Mic}_{\alpha g}\text{-cl.}$ sets and the corresponding Mic-cl. sets, $\text{Mic}_{\alpha}\text{-cl.}$ sets, and $\text{Mic}_g\text{-cl.}$ sets. A $\text{Micro}_{\alpha}T_b\text{-space}$ is the strongest among the four in the present hierarchy because every $\text{Mic}_{\alpha g}\text{-cl.}$ set is Micro closed. A $\text{Micro}_{1/2}T_{\alpha}\text{-space}$ requires every $\text{Mic}_{\alpha g}\text{-cl.}$ set to be $\text{Mic}_{\alpha}\text{-cl.}$, while a $\text{Micro}_{\alpha}T_d\text{-space}$ requires every $\text{Mic}_{\alpha g}\text{-cl.}$ set to be $\text{Mic}_g\text{-closed.}$ The $\text{Micro}_{\alpha}\text{-space}$ is different in nature: it compares $\text{Mic}_{\alpha}\text{-cl.}$ sets directly with Mic-cl. sets and therefore does not control all $\text{Mic}_{\alpha g}\text{-cl.}$ sets.

Proposition 4.1. *Every $\text{Micro}_{\alpha}T_b\text{-space}$ is a $\text{Micro}_{\alpha}\text{-space}$ but not conversely.*

Proof. Let D be $\text{Mic}_{\alpha}\text{-cl.}$ in Δ . By Result 2.1 (4) and Δ is a $\text{Micro}_{\alpha}T_b\text{-space}$, D is Mic-cl. in Δ . Hence Δ is a $\text{Micro}_{\alpha}\text{-space}$.

Example 4.1. Let $\Delta = \{e, g, h, t\}, \Delta/R = \{\{e, g\}, \{h\}, \{t\}\}$. Let $X^* = \{h, t\} \subseteq \Delta$. Then $\tau_R(X^*) = \{\Delta, \phi, \{h, t\}\}$. Let $\mu = \{g\} \notin \tau_R(X^*)$. Then $\mu_R(X^*) = \{\phi, \{g\}, \{h, t\}, \{g, h, t\}, \Delta\}$. Here Δ is a $\text{Micro}_{\alpha}\text{-space}$ but not a $\text{Micro}_{\alpha}T_b\text{-space}$, since the subset $\{e, h\}$ is $\text{Mic}_{\alpha g}\text{-cl.}$ but not Mic-cl. in Δ .

Proposition 4.2. *Every $\text{Micro}_{\alpha}T_b\text{-space}$ is a $\text{Micro}_{1/2}T_{\alpha}\text{-space}$.*

Proof. Let D be $\text{Mic}_{\alpha g}\text{-cl.}$ in Δ . Since Δ is a $\text{Micro}_{\alpha}T_b\text{-space}$, D is Mic-cl. in Δ . By Result 2.1 (1), D is $\text{Mic}_{\alpha}\text{-cl.}$ in Δ . Hence Δ is a $\text{Micro}_{1/2}T_{\alpha}\text{-space}$.

Proposition 4.3. *Every $\text{Micro}_{\alpha}T_b\text{-space}$ is a $\text{Micro}_{\alpha}T_d\text{-space}$ but not conversely.*

Proof. Let D be $\text{Mic}_{\alpha g}\text{-cl.}$ in Δ . Since Δ is a $\text{Micro}_{\alpha}T_b\text{-space}$, D is Mic-cl. in Δ . By Result 2.1 (2), D is $\text{Mic}_g\text{-cl.}$ in Δ . Hence Δ is a $\text{Micro}_{\alpha}T_d\text{-space}$.

Example 4.2. Let $\Delta = \{e, f, h, j\}, \Delta/R = \{\{e\}, \{h\}, \{f, j\}\}$. Let $X^* = \{e, f\} \subseteq \Delta$. Then $\tau_R(X^*) = \{\Delta, \phi, \{e\}, \{e, f, j\}, \{f, j\}\}$. Let $\mu = \{h\} \notin \tau_R(X^*)$. Then $\mu_R(X^*) = \{\phi, \{e\}, \{h\}, \{e, h\}, \{f, j\}, \{e, f, j\}, \{f, h, j\}, \Delta\}$. Here Δ is a $\text{Micro}_{\alpha}T_d\text{-space}$ but not a $\text{Micro}_{\alpha}T_b\text{-space}$, since the subset $\{h, j\}$ is $\text{Mic}_{\alpha g}\text{-cl.}$, but not Mic-cl. in Δ .

Proposition 4.4. *Every $\text{Micro}_{\alpha}T_b\text{-space}$ is a $\text{Micro}_{1/2}T_{\alpha}\text{-space}$.*

Proof. Let D be $\text{Mic}_g\text{-cl.}$ in Δ . By Result 2.1 (5), D is $\text{Mic}_{\alpha g}\text{-cl.}$ and since Δ is a $\text{Micro}_{\alpha}T_b\text{-space}$, D is Mic-cl. in Δ . Hence Δ is a $\text{Micro}_{1/2}T_{\alpha}\text{-space}$.

Proposition 4.5. *If Δ is a $\text{Micro}_{\alpha}T_b\text{-space}$ then $\text{Mic-}\alpha g\text{-cl}(E) = \text{Mic-}\alpha\text{cl}(E)$ for each subset E of Δ .*

Proof. Assume that Δ is a $\text{Micro}_{\alpha}T_b\text{-space}$ and Result 2.1 [1], $\text{Mic-}\alpha gC(\Delta) = \text{Mic-}\alpha C(\Delta)$. Hence $\text{Mic-}\alpha g\text{-cl}(E) = \text{Mic-}\alpha\text{cl}(E)$ for each subset E of Δ .

Theorem 4.6. *If Δ is $\text{Micro}_{\alpha}T_b$, then $\forall x \in \Delta$ either $\{x\}$ is a Mic-cl. set or Mic-op. set.*

Proof. Let $x \in \Delta$ and $\{x\}$ is a Mic-cl. set in Δ . By Lemma 2.2, $\Delta \setminus \{x\}$ is a $\text{Mic}_{\alpha g}\text{-}$

cl. set in Δ . Since Δ is a Micro ${}_{\alpha}T_b$ -space, $\Delta \setminus \{x\}$ is *Mic-cl.* in Δ or equivalently $\{x\}$ is *Mic-op.* in Δ .

Theorem 4.7. For a space $(\Delta, \tau_R(X^*), \mu_R(X^*))$ the following conditions are equivalent

(i) Δ is a Micro ${}_{1/2}T_{\alpha}$ -space.

(ii) For each $x \in \Delta$, $\{x\}$ is either a *Mic $_{\alpha}$ -op.* set or *Mic-cl.* set.

Proof. (i) $\Rightarrow$ (ii) Let $x \in \Delta$ and suppose $\{x\}$ is a *Mic-cl.* set in Δ . By Lemma 2.2, $\Delta \setminus \{x\}$ is *Mic $_{\alpha g}$ -cl.* in Δ . By (i), $\Delta \setminus \{x\}$ is *Mic $_{\alpha}$ -cl.* in Δ or equivalently $\{x\}$ is *Mic $_{\alpha}$ -op.* in Δ .

(ii) $\Rightarrow$ (i) Let D be *Mic $_{\alpha g}$ -cl.* in Δ and $x \in \text{Mic-}\alpha\text{cl}(D)$. We show that $x \in D$ for the following two cases.

Case 1: Assume that $\{x\}$ is *Mic $_{\alpha}$ -open.* Then $\Delta \setminus \{x\}$ is *Mic $_{\alpha}$ -closed.* If $x \notin D$, then $D \subseteq \Delta \setminus \{x\}$. Since $x \in \text{Mic-}\alpha\text{cl}(D)$, we have $x \in \Delta \setminus \{x\}$, which is a contradiction. Hence $x \in D$.

Case 2: Assume that $\{x\}$ is *Mic-cl.* and $x \notin D$. Then $\text{Mic-}\alpha\text{cl}(D) - D$ contains a *Mic-cl.* set $\{x\}$. This contradicts Lemma 2.1. Therefore $x \in D$.

Theorem 4.8. If Δ is a Micro ${}_{\alpha}T_d$ -space, then for each $x \in \Delta$ either $\{x\}$ is a *Mic-cl.* set or *Mic $_g$ -op.* set.

Proof. Let $x \in \Delta$ and suppose $\{x\}$ is a *Mic-cl.* set in Δ . By Lemma 2.2, $\Delta \setminus \{x\}$ is *Mic $_{\alpha g}$ -cl.* in Δ . Since Δ is a Micro ${}_{\alpha}T_d$ -space, $\Delta \setminus \{x\}$ is *Mic $_g$ -cl.* in Δ or equivalently $\{x\}$ is *Mic $_g$ -op.* in Δ .

Theorem 4.9. If Δ is both Micro ${}_{1/2}T_{\alpha}$ -space and Micro α -space, then it is a Micro ${}_{\alpha}T_b$ -space.

Proof. Let D be *Mic $_{\alpha g}$ -cl.* in Δ . Since Δ is a Micro ${}_{1/2}T_{\alpha}$ -space, D is *Mic $_{\alpha}$ -cl.* and since Δ is a Micro α -space, D is *Mic-cl.* in Δ . Hence Δ is a Micro ${}_{\alpha}T_b$ -space.

Proposition 4.10. Micro ${}_{\alpha}T_d$ -space and Micro ${}_{1/2}T_{\alpha}$ -space are independent.

Proof. Assume that Δ is a Micro ${}_{\alpha}T_d$ -space. Let D be *Mic $_{\alpha g}$ -cl.* in Δ . Since Δ is a Micro ${}_{\alpha}T_d$ -space, D is a *Mic $_g$ -cl.* set. Since *Mic $_g$ -cl.* sets need not be a *Mic $_{\alpha}$ -cl.* set, D is not a *Mic $_{\alpha}$ -cl.* set. Therefore Δ is not a Micro ${}_{1/2}T_{\alpha}$ -space.

Let D be *Mic $_{\alpha g}$ -cl.* in Δ . Since Δ is Micro ${}_{1/2}T_{\alpha}$ -space, D is a *Mic $_{\alpha}$ -cl.* set. Since *Mic $_{\alpha}$ -cl.* sets need not be a *Mic $_g$ -cl.* set, D is not a *Mic $_g$ -cl.* set. Therefore Δ is not a Micro ${}_{\alpha}T_d$ -space.

Remark 4.1. (i) Micro α -space and Micro $T_{1/2}$ -space are independent.

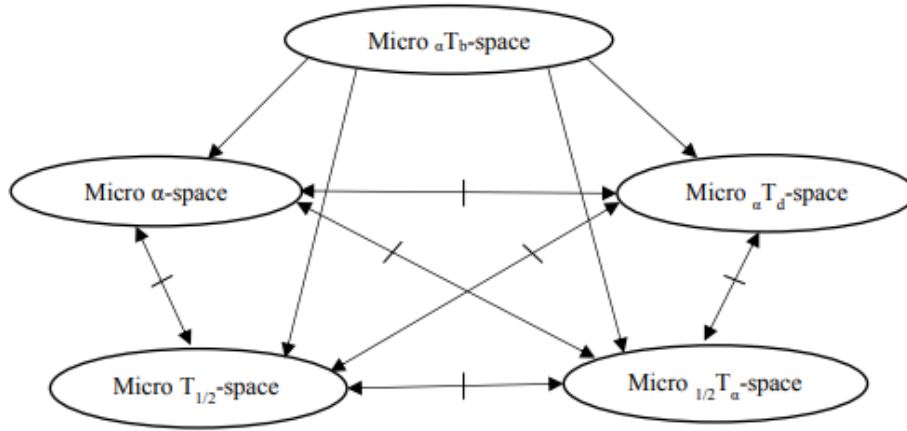
(ii) Micro α -space and Micro ${}_{1/2}T_{\alpha}$ -space are independent.

(iii) Micro α -space and Micro ${}_{\alpha}T_d$ -space are independent.

(iv) Micro $T_{1/2}$ -space and Micro ${}_{\alpha}T_d$ -space are independent.

(v) Micro $T_{1/2}$ -space and Micro ${}_{1/2}T_\alpha$ -space are independent.

Remark 4.2. From the above results, we have



5. Micro αg -continuous Maps

Definition 5.1. A map $v: (\Delta, \tau_R(X^*), \mu_R(X^*)) \rightarrow (\Omega, \tau_R(Y^*), \mu_R(Y^*))$ is called Micro αg -continuous (briefly $Mic_{\alpha g}$ -cont.) if $v^{-1}(P)$ is $Mic_{\alpha g}$ -cl. in Δ for every Mic -cl. set P in Ω .

Example 5.1. Let $\Delta = \Omega = \{x, l, w, h, i\}$, $\Delta/R = \{\{x, l\}, \{w\}, \{h\}, \{i\}\}$. Let $X^* = \{x, l, w, h, i\} \subseteq \Delta$. Then $\tau_R(X^*) = \{\Delta, \phi\}$. Let $\mu = \{i\} \notin \tau_R(X^*)$. Then $\mu_R(X^*) = \{\phi, \{i\}, \Delta\}$. Let $\Omega/R = \{\{x\}, \{l\}, \{w\}, \{h, i\}\}$. Let $Y^* = \{w, h\} \subseteq \Omega$. Then $\tau_R(Y^*) = \{\phi, \{w\}, \{w, h, i\}, \{h, i\}, \Omega\}$. Let $\mu = \{l\} \notin \tau_R(Y^*)$. Then $\mu_R(Y^*) = \{\phi, \{l\}, \{w\}, \{l, w\}, \{h, i\}, \{l, h, i\}, \{w, h, i\}, \{l, w, h, i\}, \Omega\}$. Let $v: (\Delta, \tau_R(X^*), \mu_R(X^*)) \rightarrow (\Omega, \tau_R(Y^*), \mu_R(Y^*))$ be a map defined by $v(x) = i$, $v(l) = x$, $v(w) = l$, $v(h) = w$, $v(i) = h$. Therefore, for every Mic -cl. set P in Ω , $v^{-1}(P)$ is $Mic_{\alpha g}$ -cl. in Δ . Hence v is a $Mic_{\alpha g}$ -cont. map.

Theorem 5.1. A map $v: \Delta \rightarrow \Omega$ is $Mic_{\alpha g}$ -cont. iff $v^{-1}(P)$ is $Mic_{\alpha g}$ -op. in Δ for each Mic -op. set P in Ω .

Proof. (Necessity) Let P be any Mic -op. in Ω . Then $\Omega \setminus P$ is a Mic -cl. set in Ω . By hypothesis, $v^{-1}(\Omega \setminus P) = \Delta \setminus v^{-1}(P)$ is $Mic_{\alpha g}$ -cl. in Δ . Therefore, $v^{-1}(P)$ is $Mic_{\alpha g}$ -op. in Δ .

(Sufficiency) Let Q be Mic -cl. in Ω . Then $\Omega \setminus Q$ is a Mic -op. set in Ω . By hypothesis, $v^{-1}(\Omega \setminus Q) = \Delta \setminus v^{-1}(Q)$ is a $Mic_{\alpha g}$ -op. set in Δ . Therefore, $v^{-1}(Q)$ is

$Mic_{\alpha g}$ -cl. in Δ . Therefore v is a $Mic_{\alpha g}$ -cont. map.

Proposition 5.2. Every Mic -cont. map $v: \Delta \rightarrow \Omega$ is a $Mic_{\alpha g}$ -cont. map but not conversely.

Proof. Let P be Mic -cl. in Ω . $v^{-1}(P)$ is Mic -cl. in Δ as v is a Mic -cont. map. By Result 2.1(3), $v^{-1}(P)$ is $Mic_{\alpha g}$ -cl. in Δ . Hence v is a $Mic_{\alpha g}$ -cont. map.

Example 5.2. Let $\Delta = \Omega = \{h, w, i, q\}$, $\Delta/R = \{\{h\}, \{w\}, \{i, q\}\}$. Let $X^* = \{w\} \subseteq \Delta$. Then $\tau_R(X^*) = \{\Delta, \phi, \{w\}\}$. Let $\mu = \{h, w\} \notin \tau_R(X^*)$. Then $\mu_R(X^*) = \{\phi, \{w\}, \{h, w\}, \Delta\}$. Let $\Omega/R = \{\{h, w\}, \{i\}, \{q\}\}$. Let $Y^* = \{i, q\} \subseteq \Omega$. Then $\tau_R(Y^*) = \{\Omega, \phi, \{i, q\}\}$. Let $\mu = \{w\} \notin \tau_R(Y^*)$. Then $\mu_R(Y^*) = \{\phi, \{w\}, \{i, q\}, \{w, i, q\}, \Omega\}$. Then v is a $Mic_{\alpha g}$ -cont. map when considered as $v(h) = q, v(w) = i, v(i) = w, v(q) = h$, but not a Mic -cont. map, since for the Mic -cl. set $\{h\}$ in Ω , $v^{-1}(\{h\}) = \{q\}$ is not Mic -cl. in Δ .

Proposition 5.3. Every Mic_{α} -cont. map $v: \Delta \rightarrow \Omega$ is a $Mic_{\alpha g}$ -cont. map but not conversely.

Proof. Let P be Mic -cl. in Ω . $v^{-1}(P)$ is Mic_{α} -cl. in Δ as v is a Mic_{α} -cont. map. By Result 2.1(4), $v^{-1}(P)$ is $Mic_{\alpha g}$ -cl. in Δ . Hence v is a $Mic_{\alpha g}$ -cont. map.

Example 5.3. Let $\Delta = \Omega = \{l, h, o, k\}$, $\Delta/R = \{\{l\}, \{h\}, \{o, k\}\}$. Let $X^* = \{l\} \subseteq \Delta$. Then $\tau_R(X^*) = \{\Delta, \phi, \{l\}\}$. Let $\mu = \{k\} \notin \tau_R(X^*)$. Then $\mu_R(X^*) = \{\phi, \{l\}, \{k\}, \{l, k\}, \Delta\}$. Let $\Omega/R = \{\{l, h\}, \{o\}, \{k\}\}$. Let $Y^* = \{l, h\} \subseteq \Omega$. Then $\tau_R(Y^*) = \{\Omega, \phi, \{l, h\}\}$. Let $\mu = \{o\} \notin \tau_R(Y^*)$. Then $\mu_R(Y^*) = \{\phi, \{o\}, \{l, h\}, \{l, h, o\}, \Omega\}$. Then v is a $Mic_{\alpha g}$ -cont. map when considered as $v(l) = l, v(h) = h, v(o) = k, v(k) = o$, but not a Mic_{α} -cont. map, since for the Mic -cl. set $\{o, k\}$ in Ω , $v^{-1}(\{o, k\}) = \{o, k\}$ is not Mic_{α} -cl. in Δ .

Proposition 5.4. Every Mic_g -cont. map $v: \Delta \rightarrow \Omega$ is a $Mic_{\alpha g}$ -cont. map but not conversely.

Proof. Let P be Mic -cl. in Ω . $v^{-1}(P)$ is Mic_g -cl. in Δ as v is a Mic_g -cont. map. By Result 2.1(5), $v^{-1}(P)$ is $Mic_{\alpha g}$ -cl. in Δ . Hence v is a $Mic_{\alpha g}$ -cont. map.

Example 5.4. Let $\Delta = \Omega = \{l, i, t, w\}$, $\Delta/R = \{\{l\}, \{t\}, \{i, w\}\}$. Let $X^* = \{i, w\} \subseteq \Delta$. Then $\tau_R(X^*) = \{\Delta, \phi, \{i, w\}\}$. Let $\mu = \{i\} \notin \tau_R(X^*)$. Then $\mu_R(X^*) = \{\phi, \{i\}, \{i, w\}, \Delta\}$. Let $\Omega/R = \{\{l\}, \{i\}, \{t\}, \{w\}\}$. Let $Y^* = \{t\} \subseteq \Omega$. Then $\tau_R(Y^*) = \{\Omega, \phi, \{t\}\}$. Let $\mu = \{l, w\} \notin \tau_R(Y^*)$. Then $\mu_R(Y^*) = \{\phi, \{t\}, \{l, w\}, \{l, t, w\}, \Omega\}$. Then v is a $Mic_{\alpha g}$ -cont. map when considered as $v(l) = l, v(i) = w, v(t) = t, v(w) = i$, but not a Mic_g -cont. map, since for the Mic -cl. set $\{i\}$ in Ω , $v^{-1}(\{i\}) = \{w\}$ is not Mic_g -cl. in Δ .

Remark 5.1. Micro Mic_S -cont. maps and $Mic_{\alpha g}$ -cont. maps are independent.

Example 5.5. Let $\Delta = \Omega = \{k, f, c, r\}, \Delta/R = \{\{k\}, \{f, c\}, \{r\}\}$. Let $X^* = \{k, f\} \subseteq \Delta$. Then $\tau_R(X^*) = \{\Delta, \phi, \{k\}, \{k, f, c\}, \{f, c\}\}$. Let $\mu = \{r\} \notin \tau_R(X^*)$. Then $\mu_R(X^*) = \{\phi, \{k\}, \{r\}, \{k, r\}, \{f, c\}, \{k, f, c\}, \{f, c, r\}, \Delta\}$. Let $\Omega/R = \{\{k, f\}, \{c, r\}\}$. Let $Y^* = \{k, f, c\} \subseteq \Omega$. Then $\tau_R(Y^*) = \{\Omega, \phi, \{k, f\}\}$. Let $\mu = \{c\} \notin \tau_R(Y^*)$. Then $\mu_R(Y^*) = \{\phi, \{c\}, \{k, f\}, \{k, f, c\}, \Omega\}$. Then v is a $Mic_{\alpha g}$ -cont. map when considered as $v(k) = r, v(f) = c, v(c) = k, v(r) = f$, but not a Mic_S -cont. map, since for the Mic -cl. set $\{k, f, r\}$ in Ω , $v^{-1}(\{k, f, r\}) = \{k, c, r\}$ is $Mic_{\alpha g}$ -cl. but not Mic_S -cl. in Δ .

Example 5.6. Let $\Delta = \Omega = \{r, o, c, l\}, \Delta/R = \{\{r, o\}, \{c, l\}\}$. Let $X^* = \{r, o, c\} \subseteq \Delta$. Then $\tau_R(X^*) = \{\Delta, \phi, \{r, o\}\}$. Let $\mu = \{c\} \notin \tau_R(X^*)$. Then $\mu_R(X^*) = \{\phi, \{c\}, \{r, o\}, \{r, o, c\}, \Delta\}$. Let $\Omega/R = \{\{r\}, \{o\}, \{c, l\}\}$. Let $Y^* = \{o\} \subseteq \Omega$. Then $\tau_R(Y^*) = \{\Omega, \phi, \{o\}\}$. Let $\mu = \{r, o\} \notin \tau_R(Y^*)$. Then $\mu_R(Y^*) = \{\phi, \{o\}, \{r, o\}, \Omega\}$. Then v is a Mic_S -cont. map when considered as $v(r) = c, v(o) = l, v(c) = o, v(l) = r$, but not a $Mic_{\alpha g}$ -cont. map, since for the Mic -cl. set $\{c, l\}$ in Ω , $v^{-1}(\{c, l\}) = \{r, o\}$ is Mic_S -cl. but not $Mic_{\alpha g}$ -cl. in Δ .

Remark 5.2. Mic_P -cont. maps and $Mic_{\alpha g}$ -cont. maps are independent.

Example 5.7. Let $\Delta = \Omega = \{i, k, s, h\}, \Delta/R = \{\{i\}, \{s\}, \{k, h\}\}$. Let $X^* = \{k, h\} \subseteq \Delta$. Then $\tau_R(X^*) = \{\Delta, \phi, \{k, h\}\}$. Let $\mu = \{k\} \notin \tau_R(X^*)$. Then $\mu_R(X^*) = \{\phi, \{k\}, \{k, h\}, \Delta\}$. Let $\Omega/R = \{\{i, k\}, \{s, h\}\}$. Let $Y^* = \{i, k, s\} \subseteq \Omega$. Then $\tau_R(Y^*) = \{\Omega, \phi, \{i, k\}\}$. Let $\mu = \{s\} \notin \tau_R(Y^*)$. Then $\mu_R(Y^*) = \{\phi, \{s\}, \{i, k\}, \{i, k, s\}, \Omega\}$. Then v is a $Mic_{\alpha g}$ -cont. map when considered as $v(i) = s, v(k) = i, v(s) = h, v(h) = k$, but not a Mic_P -cont. map, since for the Mic -cl. set $\{i, k, h\}$ in Ω , $v^{-1}(\{i, k, h\}) = \{k, s, h\}$ is $Mic_{\alpha g}$ -cl. but not Mic_P -cl. in Δ .

Example 5.8. Let $\Delta = \Omega = \{h, i, r, w\}, \Delta/R = \{\{h\}, \{i\}, \{r, w\}\}$. Let $X^* = \{i\} \subseteq \Delta$. Then $\tau_R(X^*) = \{\Delta, \phi, \{i\}\}$. Let $\mu = \{h, w\} \notin \tau_R(X^*)$. Then $\mu_R(X^*) = \{\phi, \{i\}, \{h, w\}, \{h, i, w\}, \Delta\}$. Let $\Omega/R = \{\{h\}, \{i\}, \{r, w\}\}$. Let $Y^* = \{i\} \subseteq \Omega$. Then $\tau_R(Y^*) = \{\Omega, \phi, \{i\}\}$. Let $\mu = \{i, r\} \notin \tau_R(Y^*)$. Then $\mu_R(Y^*) = \{\phi, \{i\}, \{i, r\}, \Omega\}$. Then v is a Mic_P -cont. map when considered as identity map, but not a $Mic_{\alpha g}$ -cont. map, since for the Mic -cl. set $\{h, w\}$ in Ω , $v^{-1}(\{h, w\}) = \{h, w\}$ is Mic_P -cl. but not $Mic_{\alpha g}$ -cl. in Δ .

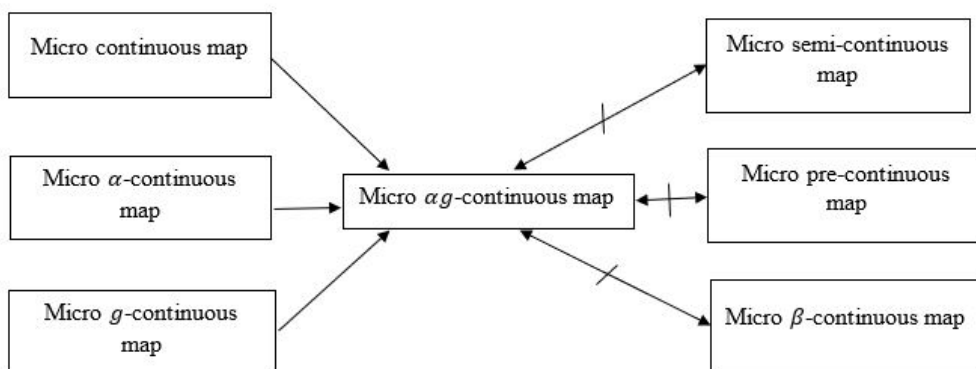
Remark 5.3. Mic_β -cont. maps and $Mic_{\alpha g}$ -cont. maps are independent.

Example 5.9. Let $\Delta = \Omega = \{l, w, r, d\}, \Delta/R = \{\{l\}, \{w, r\}, \{d\}\}$. Let $X^* = \{l, w\} \subseteq \Delta$. Then $\mu_R(X^*) = \{\Delta, \phi, \{l\}, \{l, w, r\}, \{w, r\}\}$. Let $\mu = \{d\} \notin \tau_R(X^*)$. Then $\mu_R(X^*) = \{\phi, \{l\}, \{d\}, \{l, d\}, \{w, r\}, \{l, w, r\}, \{w, r, d\}, \Delta\}$. Let $\Omega/R = \{\{l, w\}, \{r, d\}\}$. Let $Y^* = \{l, w, r\} \subseteq \Omega$. Then $\tau_R(Y^*) = \{\Omega, \phi, \{l, w\}\}$. Let $\mu = \{r\} \notin$

$\tau_R(Y^*)$. Then $\mu_R(Y^*) = \{\phi, \{r\}, \{l, w\}, \{l, w, r\}, \Omega\}$. Then v is a Mic_β -cont. map when considered as identity map, but not a $Mic_{\alpha g}$ -cont. map, since for the Mic -cl. set $\{d\}$ in Ω , $v^{-1}(\{d\}) = \{d\}$ is Mic_β -cl. but not $Mic_{\alpha g}$ -cl. in Δ .

Example 5.10. Let $\Delta = \Omega = \{m, u, k, p\}$, $\Delta/R = \{\{m\}, \{u\}, \{k, p\}\}$. Let $X^* = \{u\} \subseteq \Delta$. Then $\tau_R(X^*) = \{\Delta, \phi, \{u\}\}$. Let $\mu = \{u, k\} \notin \tau_R(X^*)$. Then $\mu_R(X^*) = \{\phi, \{u\}, \{u, k\}, \Delta\}$. Let $\Omega/R = \{\{m\}, \{u\}, \{k, p\}\}$. Let $Y^* = \{u\} \subseteq \Omega$. Then $\tau_R(Y^*) = \{\Omega, \phi, \{u\}\}$. Let $\mu = \{m, p\} \notin \tau_R(Y^*)$. Then $\mu_R(Y^*) = \{\phi, \{u\}, \{m, p\}, \{m, u, p\}, \Omega\}$. Then v is a $Mic_{\alpha g}$ -cont. map when considered as $v(m) = k, v(u) = u, v(k) = p, v(p) = m$, but not a Mic_β -cont. map, since for the Mic -cl. set $\{u, k\}$ in Ω , $v^{-1}(\{u, k\}) = \{m, u\}$ is $Mic_{\alpha g}$ -cl. but not Mic_β -cl. in Δ .

Remark 5.4. The dependency and independency relations between various types of Mic -cont. maps and $Mic_{\alpha g}$ -cont. maps are given in the following diagram,



Theorem 5.5. If $v: \Delta \rightarrow \Omega$ is a $Mic_{\alpha g}$ -cont. map then $v(Mic-\alpha g-cl(D)) \subseteq Mic-cl(v(D))$ for every $D \subseteq \Delta$.

Proof. Let $D \subseteq \Delta$. Then $Mic-cl(v(D))$ is Mic -cl. in Ω . By hypothesis, $v^{-1}(Mic-cl(v(D)))$ is $Mic_{\alpha g}$ -cl. in Δ . Since $v(D) \subseteq Mic-cl(v(D))$, $D \subseteq v^{-1}(v(D)) \subseteq v^{-1}(Mic-cl(v(D)))$. Hence $v^{-1}(Mic-cl(v(D)))$ is a $Mic_{\alpha g}$ -cl. set containing D . By definition of $Mic-\alpha g$ -closure, $Mic-\alpha g-cl(D) \subseteq v^{-1}(Mic-cl(v(D)))$ which implies that $v(Mic-\alpha g-cl(D)) \subseteq Mic-cl(v(D))$.

Corollary 5.1. Let $v: \Delta \rightarrow \Omega$ be a Mic -cont. map. Then for every subset D of Δ , $v(Mic-\alpha g-cl(D)) \subseteq Mic-cl(v(D))$.

Proof. Follows from Proposition 5.2 and by Theorem 5.5.

Remark 5.5. If $v: \Delta \rightarrow \Omega$ is $Mic_{\alpha g}$ -cont. then $v(Mic-\alpha g-cl(D))$ is not necessarily equal to $Mic-cl(v(D))$ for every subset D of Δ .

Example 5.11. In Example 5.1, take $D = \{x, l\} \subseteq \Omega$. Then $v(Mic-\alpha g-cl(D)) = v(\{x, l\}) = \{x, i\}$. But $Mic-cl(v(D)) = Mic-cl(\{x, i\}) = \{x, h, i\}$. Hence $v(Mic-\alpha g-cl(D)) \neq Mic-cl(v(D))$.

Theorem 5.6. If $v: \Delta \rightarrow \Omega$ is $Mic_{\alpha g}$ -cont. and $D \subseteq \Delta$ such that $v(Mic-\alpha g-cl(D)) \subseteq Mic-cl(v(D))$ then the inverse image of every $Mic-cl.$ set in Ω is $Mic_{\alpha g}-cl.$ in Δ .

Proof. Let P be $Mic-cl.$ in Ω and let $D = v^{-1}(P)$. Then $v(D) = v(v^{-1}(P)) \subseteq P$. Let $x \in Mic-\alpha g-cl(D)$ then $v(x) \in v(Mic-\alpha g-cl(D)) \subseteq Mic-cl(v(D)) \subseteq Mic-cl(P) = P$. Hence $x \in v^{-1}(P) = D$. Thus $Mic-\alpha g-cl(D) \subseteq D$. Therefore D is $Mic_{\alpha g}-cl.$ in Δ .

Theorem 5.7. If $v: \Delta \rightarrow \Omega$ is a $Mic_{\alpha g}$ -cont. map then for every $x \in \Delta$ and each $Mic-op.$ set T in Ω containing $v(x)$, $\exists$ a $Mic_{\alpha g}-op.$ set S in Δ containing x such that $v(S) \subseteq T$.

Proof. Let $x \in \Delta$ and $Mic-op.$ set T in Ω containing $v(x)$. Then by hypothesis, $v^{-1}(T)$ is a $Mic_{\alpha g}-op.$ set containing x . Let $S = v^{-1}(T)$ then $v(S) = v(v^{-1}(T)) \subseteq T$.

The following theorem is the main characterization result of this section. It connects a local neighbourhood condition, a direct-image closure condition, and an inverse-image closure condition.

Theorem 5.8. Let $v: (\Delta, \tau_R(X^*), \mu_R(X^*)) \rightarrow (\Omega, \tau_R(Y^*), \mu_R(Y^*))$ be a map from Δ into Ω . Then the followings are equivalent

1. For each point x in Δ and each $Mic-op.$ set T in Ω containing $v(x)$, $\exists$ a $Mic_{\alpha g}-op.$ set S in Δ containing x such that $v(S) \subseteq T$.
2. For every subset D of Δ , $v(Mic-\alpha g-cl(D)) \subseteq Mic-cl(v(D))$.
3. For every subset E of Ω , $Mic-\alpha g-cl(v^{-1}(E)) \subseteq v^{-1}(Mic-cl(E))$.

Proof. (1) $\Rightarrow$ (2): Suppose that (1) holds and let $y \in v(Mic-\alpha g-cl(D))$. Then $y = v(x)$ for some $x \in Mic-\alpha g-cl(D) \subseteq \Delta$. Let T be any $Mic-op.$ set in Ω containing $v(x)$. Then by (1), $\exists$ a $Mic_{\alpha g}-op.$ set S in Δ containing x such that $v(S) \subseteq T$. By Theorem 2.3, we have $S \cap D \neq \phi$. Therefore $v(S \cap D) \neq \phi$, implies that $T \cap v(D) \neq \phi$. Hence $y = v(x) \in Mic-cl(v(D))$. Therefore $v(Mic-\alpha g-cl(D)) \subseteq Mic-cl(v(D))$.

(2) $\Rightarrow$ (1): Suppose that (2) holds and let $x \in \Delta$ and let T be any $Mic-op.$ set in Ω containing $v(x)$. Let $D = v^{-1}(T^c)$. Then $x \notin D$. By (2), $v(Mic-\alpha g-cl(D)) \subseteq$

$\text{Mic-cl}(v(D)) \subseteq \text{Mic-cl}(v(v^{-1}(T^c))) \subseteq \text{Mic-cl}(T^c) = T^c$. Therefore $v^{-1}(v(\text{Mic-}\alpha\text{g-cl}(D))) \subseteq v^{-1}(T^c)$ which implies that $\text{Mic-}\alpha\text{g-cl}(D) \subseteq v^{-1}(T^c) = D$. Hence $D = \text{Mic-}\alpha\text{g-cl}(D)$. Since $x \notin D$, $x \notin \text{Mic-}\alpha\text{g-cl}(D)$ then by Theorem 2.3, $\exists$ a $\text{Mic-}\alpha\text{g-op.}$ set S containing x such that $S \cap D = \emptyset$ and hence $v(S) \subseteq v(D^c) \subseteq T$.

(2) $\Rightarrow$ (3): Suppose that (2) holds and let E be any subset of Ω . Replacing D by $v^{-1}(E)$ in (2), $v(\text{Mic-}\alpha\text{g-cl}(v^{-1}(E))) \subseteq \text{Mic-cl}(v(v^{-1}(E))) \subseteq \text{Mic-cl}(E)$. Hence $\text{Mic-}\alpha\text{g-cl}(v^{-1}(E)) \subseteq v^{-1}(\text{Mic-cl}(E))$.

(3) $\Rightarrow$ (2): Let (3) holds and let $E = v(D)$ where D is a subset of Δ . Then $\text{Mic-}\alpha\text{g-cl}(D) \subseteq \text{Mic-}\alpha\text{g-cl}(v^{-1}(E)) \subseteq v^{-1}(\text{Mic-cl}(E)) = v^{-1}(\text{Mic-cl}(v(D)))$. Hence $v(\text{Mic-}\alpha\text{g-cl}(D)) \subseteq \text{Mic-cl}(E) = \text{Mic-cl}(v(D))$.

Theorem 5.9. *Let $v: \Delta \rightarrow \Omega$ be $\text{Mic-}\alpha\text{g-cont.}$ then $v^{-1}(\text{Mic-int}(E)) \subseteq \text{Mic-}\alpha\text{g-int}(v^{-1}(E))$ for every subset $E \subseteq \Omega$.*

Proof. Let $E \subseteq \Omega$. Then $\text{Mic-int}(E)$ is Mic-op. in Ω . By hypothesis, $v^{-1}(\text{Mic-int}(E))$ is $\text{Mic-}\alpha\text{g-op.}$ in Δ . Hence $\text{Mic-}\alpha\text{g-int}(v^{-1}(\text{Mic-int}(E))) = v^{-1}(\text{Mic-int}(E))$. Also $\text{Mic-int}(E) \subseteq E$ implies that $v^{-1}(\text{Mic-int}(E)) \subseteq v^{-1}(E)$. Therefore $\text{Mic-}\alpha\text{g-int}(v^{-1}(\text{Mic-int}(E))) \subseteq \text{Mic-}\alpha\text{g-int}(v^{-1}(E))$. Hence $v^{-1}(\text{Mic-int}(E)) \subseteq \text{Mic-}\alpha\text{g-int}(v^{-1}(E))$.

Remark 5.6. *If $v: \Delta \rightarrow \Omega$ is $\text{Mic-}\alpha\text{g-cont.}$ then $v^{-1}(\text{Mic-int}(E)) \neq \text{Mic-}\alpha\text{g-int}(v^{-1}(E))$ for every subset $E \subseteq \Omega$.*

Example 5.12. In Example 5.1, take $E = \{x, l\} \subseteq \Omega$. Then $v^{-1}(\text{Mic-int}(E)) = v^{-1}(\text{Mic-int}(\{x, l\})) = v^{-1}(\{l\}) = w$. $\text{Mic-}\alpha\text{g-int}(v^{-1}(E)) = \text{Mic-}\alpha\text{g-int}(v^{-1}(\{x, l\})) = \text{Mic-}\alpha\text{g-int}(\{l, w\}) = \{l, w\}$. Hence $v^{-1}(\text{Mic-int}(E)) \neq \text{Mic-}\alpha\text{g-int}(v^{-1}(E))$.

Proposition 5.10. *Let $(\Delta, \tau_R(X^*), \mu_R(X^*))$, $(\Omega, \tau_R(Y^*), \mu_R(Y^*))$ and $(\psi, \tau_R(Z^*), \mu_R(Z^*))$ be the three Micro topological spaces. If $v: (\Delta, \tau_R(X^*), \mu_R(X^*)) \rightarrow (\Omega, \tau_R(Y^*), \mu_R(Y^*))$ is a $\text{Mic-}\alpha\text{g-cont.}$ map and $w: (\Omega, \tau_R(Y^*), \mu_R(Y^*)) \rightarrow (\psi, \tau_R(Z^*), \mu_R(Z^*))$ is a Mic-cont. map, then $(w \circ v): (\Delta, \tau_R(X^*), \mu_R(X^*)) \rightarrow (\psi, \tau_R(Z^*), \mu_R(Z^*))$ is a $\text{Mic-}\alpha\text{g-cont.}$ map.*

Proof. Let P be Mic-cl. in ψ . $w^{-1}(P)$ is Mic-cl. in Ω as w is a Mic-cont. map. By hypothesis, $(w \circ v)^{-1}(P)$ is $\text{Mic-}\alpha\text{g-cl.}$ in Δ . Therefore $w \circ v$ is a $\text{Mic-}\alpha\text{g-cont.}$ map.

Remark 5.7. *If $v: \Delta \rightarrow \Omega$ is a Mic-cont. map and $w: \Omega \rightarrow \psi$ is a $\text{Mic-}\alpha\text{g-cont.}$ map then $(w \circ v): \Delta \rightarrow \psi$ need not be a $\text{Mic-}\alpha\text{g-cont.}$ map.*

Example 5.13. Let $\Delta = \Omega = \psi = \{w, o, r, i\}$, $\Delta/R = \{\{w\}, \{o\}, \{r, i\}\}$. Let $X^* = \{o\} \subseteq \Delta$. Then $\tau_R(X^*) = \{\Delta, \phi, \{o\}\}$. Let $\mu = \{w, o\} \notin \tau_R(X^*)$. Then $\mu_R(X^*) = \{\phi, \{o\}, \{w, o\}, \Delta\}$. Let $\Omega/R = \{\{w, o\}, \{r\}, \{i\}\}$. Let $Y^* =$

$\{w\} \subseteq \Omega$. Then $\tau_R(Y^*) = \{\Omega, \phi, \{w, o\}\}$. Let $\mu = \{o\} \notin \tau_R(Y^*)$. Then $\mu_R(Y^*) = \{\phi, \{o\}, \{w, o\}, \Omega\}$. Let $\psi/R = \{\{w, o\}, \{r\}, \{i\}\}$. Let $Z^* = \{w\} \subseteq \psi$. Then $\tau_R(Z^*) = \{\psi, \phi, \{w, o\}\}$. Let $\mu = \{r\} \notin \tau_R(Z^*)$. Then $\mu_R(Z^*) = \{\phi, \{r\}, \{w, o\}, \{w, o, r\}, \psi\}$. Then the map v is a *Mic-cont.* map when considered as identity map and w is a *Mic $_{\alpha g}$ -cont.* map when considered as $w(w) = i, w(o) = o, w(r) = r, w(i) = w$, but $w \circ v$ is not *Mic $_{\alpha g}$ -cont.*, since for the *Mic-cl.* set $\{w, o, i\}$ in ψ , $(w \circ v)^{-1}(\{w, o, i\}) = v^{-1}(w^{-1}(\{w, o, i\})) = v^{-1}(\{w, o, i\}) = \{w, o, i\}$ is not *Mic $_{\alpha g}$ -cl.* in Δ .

Proposition 5.11. *If $v: \Delta \rightarrow \Omega$ is a *Mic $_{\alpha}$ -cont.* map and $w: \Omega \rightarrow \psi$ is a *Mic-cont.* map, then $(w \circ v): \Delta \rightarrow \psi$ is a *Mic $_{\alpha g}$ -cont.* map.*

Proof. Let P be *Mic-cl.* in ψ . $w^{-1}(P)$ is *Mic-cl.* in Ω as w is a *Mic-cont.* map. Since v is *Mic $_{\alpha}$ -cont.*, $(w \circ v)^{-1}(P)$ is *Mic $_{\alpha}$ -cl.* in Δ . By Result 2.1 (4), $(w \circ v)^{-1}(P)$ is *Mic $_{\alpha g}$ -cl.* in Δ . Therefore $w \circ v$ is a *Mic $_{\alpha g}$ -cont.* map.

Proposition 5.12. *If $v: \Delta \rightarrow \Omega$ is a *Mic $_g$ -cont.* map and $w: \Omega \rightarrow \psi$ is a *Mic-cont.* map, then their composition $(w \circ v): \Delta \rightarrow \psi$ is a *Mic $_{\alpha g}$ -cont.* map.*

Proof. Let P be *Mic-cl.* in ψ . $w^{-1}(P)$ is *Mic-cl.* in Ω as w is a *Mic-cont.* map. Since v is *Mic $_g$ -cont.*, $(w \circ v)^{-1}(P)$ is *Mic $_g$ -cl.* in Δ . By Result 2.1 (5), $(w \circ v)^{-1}(P)$ is *Mic $_{\alpha g}$ -cl.* in Δ . Therefore $w \circ v$ is a *Mic $_{\alpha g}$ -cont.* map.

Remark 5.8. *The composition of two *Mic-cont.* maps is a *Mic $_{\alpha g}$ -cont.* map.*

Proof. Let $v: \Delta \rightarrow \Omega$ and $w: \Omega \rightarrow \psi$ be two *Mic-cont.* maps. Let P be a *Mic-cl.* set in ψ . $w^{-1}(P)$ is *Mic-cl.* in Ω as w is a *Mic-cont.* map. Since v is *Mic-cont.*, $(w \circ v)^{-1}(P)$ is *Mic-cl.* in Δ . By Result 2.1 (3), $(w \circ v)^{-1}(P)$ is *Mic- αg -cl.* in Δ . Therefore $w \circ v$ is a *Mic $_{\alpha g}$ -cont.* map.

Remark 5.9. *The composition of two *Mic $_{\alpha g}$ -cont.* maps need not be a *Mic $_{\alpha g}$ -cont.* map.*

Example 5.14. Let $\Delta = \Omega = \psi = \{k, i, r, q\}$, $\Delta/R = \Omega/R = \psi/R = \{\{k\}, \{i\}, \{r, q\}\}$. Let $X^* = \{i\} \subseteq \Delta$, $Y^* = \{i\} \subseteq \Omega$, $Z^* = \{i\} \subseteq \psi$. Then $\tau_R(X^*) = \{\Delta, \phi, \{i\}\}$, $\tau_R(Y^*) = \{\Omega, \phi, \{i\}\}$, $\tau_R(Z^*) = \{\psi, \phi, \{i\}\}$. Let $\mu = \{k, q\} \notin \tau_R(X^*)$. Then $\mu_R(X^*) = \{\phi, \{i\}, \{k, q\}, \{k, i, q\}, \Delta\}$. Let $\mu = \{i, r\} \notin \tau_R(Y^*)$. Then $\mu_R(Y^*) = \{\phi, \{i\}, \{i, r\}, \Omega\}$. Let $\mu = \{i, q\} \notin \tau_R(Z^*)$. Then $\mu_R(Z^*) = \{\phi, \{i\}, \{i, q\}, \psi\}$. Then the maps v and w are *Mic $_{\alpha g}$ -cont.* when considered as $v(k) = i, v(i) = k, v(r) = q, v(q) = r$ and w as identity map, but $w \circ v$ is not *Mic $_{\alpha g}$ -cont.*, since for the *Mic-cl.* set $\{k, r\}$ in ψ , $(w \circ v)^{-1}(\{k, r\}) = v^{-1}(w^{-1}(\{k, r\})) = v^{-1}(\{k, r\}) = \{i, q\}$ is not *Mic $_{\alpha g}$ -cl.* in Δ .

The following proposition preserves composition of two *Mic $_{\alpha g}$ -cont.* maps is *Mic $_{\alpha g}$ -cont.* when Ω is a Micro $_{\alpha}T_b$ -space.

Proposition 5.13. *Let $v: \Delta \rightarrow \Omega$ and $w: \Omega \rightarrow \psi$ be $Mic_{\alpha g}$ -cont. maps. Then $(w \circ v): \Delta \rightarrow \psi$ is also a $Mic_{\alpha g}$ -cont. map, if Ω is a $Micro_{\alpha}T_b$ -space.*

Proof. Let P be Mic -cl. in ψ . $w^{-1}(P)$ is $Mic_{\alpha g}$ -cl. in Ω as w is a $Mic_{\alpha g}$ -cont. map. Since Ω is a $Micro_{\alpha}T_b$ -space, $w^{-1}(P)$ is Mic -cl. in Ω . $(w \circ v)^{-1}(P)$ is $Mic_{\alpha g}$ -cl. in Δ as v is a $Mic_{\alpha g}$ -cont. map. Therefore $w \circ v$ is a $Mic_{\alpha g}$ -cont. map.

Proposition 5.14. *If $v: \Delta \rightarrow \Omega$ is a $Mic_{\alpha g}$ -cont. map and $w: \Omega \rightarrow \psi$ is a Mic -cont. map, then $(w \circ v): \Delta \rightarrow \psi$ is a Mic_{α} -cont. map, if Δ is a $Micro_{1/2}T_{\alpha}$ -space.*

Proof. Let P be Mic -cl. in ψ . $w^{-1}(P)$ is Mic -cl. in Ω as w is a Mic -cont. map. $(w \circ v)^{-1}(P)$ is $Mic_{\alpha g}$ -cl. in Δ as v is a $Mic_{\alpha g}$ -cont. map. Since Δ is a $Micro_{1/2}T_{\alpha}$ -space, $(w \circ v)^{-1}(P)$ is Mic_{α} -cl. in Δ . Therefore $w \circ v$ is a Mic_{α} -cont. map.

Proposition 5.15. *If $v: \Delta \rightarrow \Omega$ is a $Mic_{\alpha g}$ -cont. map and $w: \Omega \rightarrow \psi$ is a Mic -cont. map, then $(w \circ v): \Delta \rightarrow \psi$ is a Mic_g -cont. map, if Δ is a $Micro_{\alpha}T_d$ -space.*

Proof. Let P be Mic -cl. in ψ . $w^{-1}(P)$ is Mic -cl. in Ω as w is a Mic -cont. map. $(w \circ v)^{-1}(P)$ is $Mic_{\alpha g}$ -cl. in Δ as v is a $Mic_{\alpha g}$ -cont. map. Since Δ is a $Micro_{\alpha}T_d$ -space, $(w \circ v)^{-1}(P)$ is Mic_g -cl. in Δ . Therefore $w \circ v$ is a Mic_g -cont. map.

6. Conclusion

In this paper, we introduced and studied the $Micro_{\alpha g}$ -closure, $Micro_{\alpha g}$ -interior, and $Micro_{\alpha g}$ -continuous maps in $Micro$ topological spaces. Their relationships with existing $Micro$ structures were investigated through suitable propositions and examples. We also examined the associated $Micro$ separation spaces and obtained several characterizations and composition results for $Micro_{\alpha g}$ -continuous maps. These results provide a useful framework for further studies on generalized $Micro$ topological structures and mappings. The study can be extended to $Micro$ subspace and $Micro$ product space in $Micro$ topological spaces.

References

- [1] Anandhi, R. and Balamani, N., $Micro_{\alpha}$ -Generalized Closed Sets in $Micro$ Topological Spaces, AIP Conference Proceedings, 2385 (2022), 130007; <https://doi.org/10.1063/5.0071060>.
- [2] Anandhi, R. and Balamani, N., $Micro_{\alpha g}$ -Separation Axioms, Advances and Applications in Mathematical Sciences, 21(4) (2022), 1925-1936.
- [3] Carmel Richard, Studies on Nano Topological Spaces, Ph.D. Thesis, Madurai Kamaraj University, Madurai, India, 2013.
- [4] Chandrasekar, S. and Swathi, G., $Micro_{\alpha}$ -open sets in $Micro$ Topological Spaces, International Journal of Research in advent Technology, 6 (2018), 2633-2637.

- [5] Ibrahim, H. Z., On Micro $T_{1/2}$ -Space, International Journal of Applied Mathematics, 33 (2020), 369-384.
- [6] Ibrahim, H. Z., Micro β -open sets in Micro topology, General Letters in Mathematics, 8(1) (2020), 8-15.
- [7] Ibrahim, H. Z., Micro Separation Axioms, International Journal of Analysis and Applications, 18 (2020), 572-585.
- [8] Ibrahim, H. Z., On Micro b -open sets, Asia Mathematika, 6(2) (2022), 20-32.
- [9] Reem O Rasheed and Taha H Jasim, On Micro- α -open sets and Micro- α -continuous functions in Micro Topological Spaces, J. Phys.: Conf, Ser., (2020), 1530 012061.
- [10] Sakkraveeranan Chandrasekar, On Micro Topological Spaces, Journal of New Theory, 26 (2019), 23-31.
- [11] Taha H Jasim, Saja S Mohsen, Kanayo S Eke, On Micro-generalized closed sets and Micro-generalized continuity in Micro Topological Spaces, European Journal of Pure and Applied Mathematics, 14(4) (2021), 1507-1516.

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