

**INTRODUCTION TO THE GANITA ALGORITHM IN
KR-INTUITIONISTIC FUZZY EQUIVALENCE RELATION:
DESIGN, EFFICIENCY, AND APPLICATION**

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Abstract: In recent years, Artificial Intelligence (AI) has revolutionized a wide range of fields; however, challenges related to uncertainty, computational efficiency, and reliability continue to remain. This paper introduces a novel KR-Intuitionistic Fuzzy Equivalence Relation, developed through the reconstruction and refinement of existing intuitionistic fuzzy relational models. The proposed relation incorporates a Known-Restriction (KR) under which the degree of belongingness is maintained above the degree of non-belongingness, providing a framework for handling uncertain and conflicting information in multi-compositional settings. In addition, the paper proposes the Ganita Algorithm as an efficient computational approach for reducing the computational effort associated with multi-composition of intuitionistic fuzzy relations. A detailed comparative study of fuzzy, intuitionistic fuzzy, and KR-intuitionistic fuzzy frameworks is presented, together with an analysis of the distinguishing characteristics and advantages of KR-intuitionistic fuzzy sets over conventional fuzzy and intuitionistic fuzzy approaches. The proposed framework is further examined through a potential application AI in safety, where the KR restriction can help preserve reliable associations while processing uncertain information and repeated relational compositions.

Keywords and Phrases: Intuitionistic Fuzzy Set, Intuitionistic Fuzzy Relation, Decision Making, Risk Optimization.

2020 Mathematics Subject Classification: 03B52, 90B50, 97E60.

1. Introduction

The theory of Fuzzy Sets (FSs) by Zadeh [27] has become an important mathematical tool for dealing with uncertainty and imprecision in a wide range of applications. The concept of fuzzy sets (FSs) considers only the degree of membership of an element to a set, while the degree of non-membership is implicitly dependent on the membership degree. To address this limitation, Atanassov introduced Intuitionistic Fuzzy Sets (IFSs) [7], in which the degrees of membership and non-membership are treated independently, thereby providing a more flexible framework for handling uncertainty. Following the introduction of Intuitionistic Fuzzy Sets, Atanassov [8] introduced the concept of Intuitionistic Fuzzy Relations (IFRs), thereby extending the theory to represent relationships between intuitionistic fuzzy sets. He subsequently investigated several theoretical aspects of IFSs, including their fundamental properties [6], remarks on the theory [5], new operations on intuitionistic fuzzy sets [4], and their comprehensive mathematical framework ([3, 2]). Burillo and Bustince [13] examined the basic properties of intuitionistic fuzzy relations, and in a subsequent study [14], they analysed the effect of Atanassov's operators on these relations. Liu and Wang [19] demonstrated the applicability of intuitionistic fuzzy sets to multi-criteria decision-making, while Xu and Yager [26] proposed intuitionistic fuzzy preference relations for evaluating consensus in group decision-making. Panigrahi and Nanda [21] further investigated intuitionistic fuzzy relations over intuitionistic fuzzy sets, and Kang et al. [18] studied intuitionistic fuzzy mappings and intuitionistic fuzzy equivalence relations. In addition, Ovchinnikov [20] developed numerical representations of transitive fuzzy relations, Baets et al. [10] investigated cardinality-based similarity measures, Bosteels and Kerre [12] proposed reflexivity-preserving fuzzy comparison measures, and Jayaram and Mesiar [17] studied fuzzy equivalence relations using conjunctors and implicators. In particular, Li et al. [11] studied the structures of intuitionistic fuzzy equivalence relations, including their lattice structure and associated α -cut relations, providing a more detailed structural treatment of IFRs. These contributions have established a strong theoretical foundation for intuitionistic fuzzy relations and their applications in uncertainty modelling and intelligent decision-making.

Now, this new concept of members and non-members plays a crucial role in any time of decision making, particular those with involves the role of Artificial Intelligence (AI) as it not only take into account the members but also non-members while making a decision. This particular aspect will be explored as we go on through this paper. In recent years, the rapid growth of Artificial Intelligence (AI) has increased the demand for mathematical models capable of handling uncertain and incomplete information efficiently. Reliable decision-making in AI often

requires not only identifying supporting evidence but also accounting for contradictory or missing information. This has renewed interest in intuitionistic fuzzy theory because of its ability to represent both membership and non-membership independently. Recent studies have focused on improving similarity measures, computational methods, and intelligent applications of IFSs. For example, Atanassov and De Tre [1] revisited intuitionistic fuzzy subset relations by proposing an intuitionistic evaluation method for the is subset of relationship between two crisp sets, while Wu et al. [25] proposed new similarity and distance measures with applications in pattern recognition and medical diagnosis. Patel et al. [22] further developed similarity measures for intuitionistic fuzzy sets and demonstrated their usefulness in multimodal medical image fusion. More recently, Atheeqe and Basha [9] applied intuitionistic fuzzy graph models to decision-making problems, Pratama et al. [24] introduced improved ordering and aggregation operators for circular intuitionistic fuzzy sets, and Dawa et al. [16] proposed a similarity-based three-way decision model for intuitionistic fuzzy information systems. These studies highlight the growing role of intuitionistic fuzzy theory in modern AI and intelligent decision support.

The paper is organized as follows. Section 2 presents the necessary preliminaries on Intuitionistic Fuzzy Sets (IFSs) and existing Intuitionistic Fuzzy Relations (IFRs). Section 3 introduces the proposed KR-Intuitionistic Fuzzy Equivalence Relation (KR-IFER) and examines its fundamental properties. Section 4 introduces the Ganita Algorithm and examines its computational aspects and efficiency in multi-composition. Section 5 presents a detailed comparison among crisp, fuzzy, intuitionistic fuzzy, and KR-intuitionistic fuzzy frameworks, along with a detailed analysis of their respective characteristics. Section 6 discusses the benefits and distinguishing features of the proposed KR-Intuitionistic Fuzzy Equivalence Relation. Finally, Section 7 presents an application of the proposed framework to a Virtual Safety Engineer (VSE) in the context of AI safety.

2. Intuitionistic Fuzzy Set and Relation

As given by Atanassov ([7], [8], [4]), for an universe U , a fuzzy subset I_F of U is an Intuitionistic Fuzzy Set (IFS) if it is of the form:

$I_F = \{ \langle k, \mu_{I_F}(k), \nu_{I_F}(k) \mid k \in U \rangle \}$ where $\mu_{I_F} : U \rightarrow [0, 1]$ is called the membership function and $\nu_{I_F} : U \rightarrow [0, 1]$ is called the non-membership function with the condition $\mu_{I_F} + \nu_{I_F} \leq 1 \forall k \in U$ called the Intuitionistic Condition [21]. Here, $\mu_{I_F}(x)$ and $\nu_{I_F}(x)$ are called membership degree and non-membership degree respectively. So, an Intuitionistic Fuzzy Set $I_F \in IFS(U)$ can be denoted by (μ_{I_F}, ν_{I_F}) .

If R_F be a fuzzy relation on IFS of $U \times U$ with $R_F \subseteq K \times R$ i.e. $\forall (k, r) \in U \times U$ we get $\mu_{I_F}(k, r) \leq \mu_{K \times R}(k, r)$ and $\nu_{I_F}(k, r) \geq \nu_{K \times R}(k, r)$ with $\mu_{I_F}(k, r) + \nu_{I_F}(k, r) \leq 1$,

then R_F is called Intuitionistic Fuzzy Relation (IFR) from K to R . It is denoted by $IFR(K, R)$ [21].

An intuitionistic fuzzy relation on R_F is called an intuitionistic fuzzy equivalence relation (IFER) [18] on $U \times U$ if it satisfies the following conditions:

- it is intuitionistic fuzzy reflexive, i.e. $R_F(k, k) = (1, 0) \forall k \in U$,
- it is intuitionistic fuzzy symmetric, i.e. $R_F^{-1} = R_F$ i.e. $\mu(k, r) = \mu(r, k)$ and $\nu(k, r) = \nu(r, k) \forall (k, r) \in U \times U$,
- it is intuitionistic fuzzy transitive, i.e. $R_F \circ R_F = R_F$ where $\circ$ denotes the standard *max – min* composition.

3. KR-Intuitionistic Fuzzy Equivalence Relation

This paper proposes a novel approach to Intuitionistic Fuzzy Equivalence Relation (IFER) by restructuring the conventional definitions of symmetricity and transitivity. The resulting relation is termed the KR-Intuitionistic Fuzzy Equivalence Relation, where KR stands for Known-Restriction. The formal definition of this newly introduced Intuitionistic Fuzzy Equivalence Relation is as follows.

For an IFS I_F in an universe U , an IFR R_F will be called KR-Intuitionistic Fuzzy Equivalence Relation if

- Reflexivity: $\mu_{I_F}(k, k) = 1$ and $\nu_{I_F}(k, k) = 0 \forall k \in U$.
- Restricted Symmetricity: $\mu_{I_F}(k, r) = \mu_{I_F}(r, k)$ and $\nu_{I_F}(k, r) = \nu_{I_F}(r, k)$ with $\mu_{I_F}(k, r) > \nu_{I_F}(k, r) \forall k, r \in U$.
- Restricted Transitivity: $\sup_r [\min\{\mu_{I_F}(k, r), \mu_{I_F}(r, t)\}] \leq \mu_{I_F}(k, t)$ and $\nu_{I_F}(k, t) \leq 1 - \mu_{I_F}(k, t)$ with $\mu_{I_F}(k, r) > \nu_{I_F}(k, r) \forall k, r \in U$.
The condition $\mu_{I_F}(k, r) > \nu_{I_F}(k, r)$ is called Known-Restriction (KR).

3.1. Significance of the Known-Restriction

The Known-Restriction, $\mu_{I_F}(k, r) > \nu_{I_F}(k, r) \forall k, r \in U$ mandates that membership value should always be greater than non-membership value. This condition emphasizes the prioritization of elements that are more beneficial or supportive of the structure of the set. In case of the definition of symmetry, it focuses only on trusted relationship where symmetry is only required where members dominates. In case of the definition of transitivity, it automatically determines the values of non-members aggressively assuming that non-members do not play a significant role in transitivity. The advantage of this restriction is discussed in detail in the application section later in the paper.

3.2. Crisp Reduction of the KR-Intuitionistic Fuzzy Equivalence Relation

An important property of an Intuitionistic Fuzzy Equivalence Relation (IFER) is that it can be represented by a crisp equivalence relation through an appropriate cut. Li and He [11] defined the α -cut of an intuitionistic fuzzy relation $R = (\mu_R, \nu_R)$ as

$$R_\alpha = \{(k, r) \in U \times U : \mu_R(k, r) \geq \mu_\alpha, \nu_R(k, r) \leq \nu_\alpha\}$$

where $\alpha = (\mu_\alpha, \nu_\alpha)$ is an intuitionistic fuzzy value. They established that, when R is an IFER, the corresponding cut relation is a crisp equivalence relation on U . Motivated by this cut-based characterization, we investigate the crisp reduction of the proposed KR-Intuitionistic Fuzzy Equivalence Relation.

In the proposed KR-framework, the restricted transitivity condition is formulated explicitly in terms of the membership function. Therefore, for the purpose of establishing a crisp equivalence relation, we consider the membership-based α -cut

$$R_\alpha^{KR} = \{(k, r) \in U \times U : \mu_R(k, r) \geq \alpha\}, \quad 0 \leq \alpha \leq 1$$

The non-membership function is not included as a separate condition in this cut because the proposed KR-relation already imposes the restriction

$$\mu_R(k, r) > \nu_R(k, r)$$

together with

$$\mu_R(k, r) + \nu_R(k, r) \leq 1.$$

Thus, the non-membership degree remains an essential part of the proposed intuitionistic fuzzy relation, while the crisp reduction is determined by the membership threshold for which the required reflexivity, symmetry, and restricted transitivity are directly available.

Proposition 3.1. *Let $R = (\mu_R, \nu_R)$ be a KR-Intuitionistic Fuzzy Equivalence Relation on U . For $\alpha \in [0, 1]$, define*

$$R_\alpha^{KR} = \{(k, r) \in U \times U : \mu_R(k, r) \geq \alpha\}$$

Then R_α^{KR} is a crisp equivalence relation on U .

Proof. We prove that R_α^{KR} satisfies reflexivity, symmetry, and transitivity.

Reflexivity: By the reflexivity of the KR-Intuitionistic Fuzzy Equivalence Relation,

$$\mu_R(k, k) = 1 \quad \forall k \in U$$

Since $\alpha \in [0, 1]$, it follows that

$$\mu_R(k, k) = 1 \geq \alpha.$$

Therefore,

$$(k, k) \in R_\alpha^{KR} \quad \forall k \in U.$$

Hence, R_α^{KR} is reflexive.

Symmetry: Suppose that $(k, r) \in R_\alpha^{KR}$

Then,

$$\mu_R(k, r) \geq \alpha$$

By the restricted symmetricity of R ,

$$\mu_R(k, r) = \mu_R(r, k)$$

Therefore,

$$\mu_R(r, k) \geq \alpha$$

which implies

$$(r, k) \in R_\alpha^{KR}$$

Hence, R_α^{KR} is symmetric.

Transitivity: Suppose that

$$(k, r) \in R_\alpha^{KR} \quad \text{and} \quad (r, t) \in R_\alpha^{KR}$$

Then

$$\mu_R(k, r) \geq \alpha \quad \text{and} \quad \mu_R(r, t) \geq \alpha$$

Consequently,

$$\min\{\mu_R(k, r), \mu_R(r, t)\} \geq \alpha$$

By the restricted transitivity condition of the proposed relation,

$$\mu_R(k, t) \geq \sup_{r \in U} \min\{\mu_R(k, r), \mu_R(r, t)\} \geq \alpha$$

Therefore,

$$(k, t) \in R_\alpha^{KR}$$

Hence, R_α^{KR} is transitive.

Since R_α^{KR} is reflexive, symmetric, and transitive, it is a crisp equivalence relation on U .

The above result establishes a direct connection between the proposed KR-Intuitionistic Fuzzy Equivalence Relation and its corresponding crisp equivalence structure. In particular, the intuitionistic information is preserved in the original relation through both μ_R and ν_R , while the membership-based cut provides a crisp representation suitable for classification, partitioning, or subsequent decision-making.

3.3. Concept of Multi-Composition and Elementary Relation

The Multi-Composition i.e. the composition of multiple relations allow us to perform composition of more than two relations simultaneously with the help of the Known Restriction and the Ganita Algorithm (Section 4). This is illustrated in detail in the Section 4. Each individual relation that are part of this type of composition is called Elementary Relation.

n multi-composition form: $R_{1_{IF}} \circ R_{2_{IF}} \circ R_{3_{IF}} \circ \dots \circ R_{n_{IF}}$

Here, $R_{i_{IF}}$ is an elementary relation at i^{th} -position.

The newly introduced Known-Restriction (KR) condition can be particularly advantageous in scenarios involving multi-composition, where more than two intuitionistic fuzzy relations are composed simultaneously. By ensuring that the membership degree consistently dominates the non-membership degree, the KR condition helps to maintain interpretability and consistency across multiple layers of relational composition.

4. The Ganita Algorithm

Suppose I_F is an IFS on an universe U and $\mu_{I_F}(k, r)$ and $\nu_{I_F}(k, r)$ denotes membership and non-membership grades on a KR-Intuitionistic Fuzzy Relation. Then according to the Ganita Algorithm, preference will be given to those elementary relations in a multi-composition that satisfies the Known-Restriction (KR) condition i.e. $\mu_{I_F}(k, r) > \nu_{I_F}(k, r) \forall k, r \in U$. However, if elementary relations does not satisfy the KR condition then priority will be based on a ranking system determined as Rank Value (RV) = $|\mu_{I_F}(k, r) - \nu_{I_F}(k, r)|$ whenever $\mu_{I_F}(k, r) \leq \nu_{I_F}(k, r)$. A lower rank value (RV) indicates a greater preference with $RV = 0$ being the most preferred.

4.1. An Illustration. Notation: $R_{i_{IF}} = (\mu_{i_{IF}}, \nu_{i_{IF}})$ denotes an IFR.

Consider six IFRs, $R_{1_{IF}} = (0.6, 0.4)$, $R_{2_{IF}} = (0.7, 0.3)$, $R_{3_{IF}} = (0.4, 0.6)$, $R_{4_{IF}} = (0.8, 0.2)$, $R_{5_{IF}} = (0.5, 0.5)$ and $R_{6_{IF}} = (0.9, 0.1)$ then the composition:

$R_{1_{IF}} \circ R_{2_{IF}} \circ R_{3_{IF}} \circ R_{4_{IF}} \circ R_{5_{IF}} \circ R_{6_{IF}}$
will become

$R_{6_{IF}} \circ R_{4_{IF}} \circ R_{2_{IF}} \circ R_{1_{IF}} \circ R_{5_{IF}} \circ R_{3_{IF}}$ (according to the Ganita Algorithm)

This can be further classify as a group satisfying or not satisfying the KR

condition as $(R_{6_{IF}} \circ R_{4_{IF}} \circ R_{2_{IF}} \circ R_{1_{IF}}) \circ (R_{5_{IF}} \circ R_{3_{IF}})$, which can be sub-grouped as $(R_{6_{IF}} \circ R_{4_{IF}}) \circ (R_{2_{IF}} \circ R_{1_{IF}}) \circ (R_{5_{IF}} \circ R_{3_{IF}})$.

4.2. Significance of the Ganita Algorithm

The Ganita Algorithm proves especially beneficial in multi-compositional computations, where more than two intuitionistic fuzzy relations are combined simultaneously. In such scenarios, preserving a dominance of positive evidence across successive compositions helps mitigate uncertainty accumulation and semantic drift. This is particularly useful in applications such as hierarchical clustering, and layered knowledge representation, where interpretability, transitive consistency, and robustness are critical.

5. A Detailed Comparison

Detailed Comparison: Crisp, Fuzzy and Intuitionistic Fuzzy Logic			
Aspect	Crisp Logic	Standard Fuzzy Logic	KR-Intuitionistic Fuzzy Logic
What it models?	Yes/No or single Probability (e.g.: $P = 0.6$).	Degree of Membership (e.g.: $\mu = 0.6$).	Degree of Membership(μ) and Non-Membership (ν) (e.g.: $\mu = 0.6, \nu = 0.4$).
Safety Interpretation	The component failure rate is high.	The component's failure is high to degree 0.6.	The component's failure is to degree 0.6 and non-failure to 0.4.
Ease of Expert Input	Must suggest YES/NO or give point probabilities.	Can give belief ratings mapped to μ .	Can express both belief & doubt.
Combining multiple indicators	Uses rules or Bayesian multi-plication.	Uses fuzzy AND/OR rules.	Propagates through Ganita Alogorithm.
Computational time for complex comibinations	Moderate.	High.	Low.

6. A Detailed Analysis

Based on the discussion of this paper, a model was constructed to analyse the difference among Fuzzy, Intuitionistic Fuzzy and KR-Intuitionistic Fuzzy Sets. This section gives us the overview of this detailed analysis of various aspects among Fuzzy Sets, Intuitionistic Fuzzy Sets (IF) and KR-Intuitionistic Fuzzy Sets (IF-Prep). Here, we take the help o two functions: alpha (α) and delta (Δ) to arrive at certain conclconclusion.

- **Alpha** (α): It is the preference intensity coefficient — a scaling factor that controls how much the system reinforces the membership value (μ) when it's greater than the non-membership (ν). When $\alpha = 0$, the model behaves like a normal fuzzy logic - normal fuzzy logic - no preference adjustment but as the value of the alpha (α) increases, the model trusts strong memberships more - increasing μ slightly and reducing ν . So, increase in number of compositions increases this strong membership preference levels.
- **Delta** (Δ): It is the actual preference correction value computed for each input point. It depends on the current confidence gap calculated as difference between μ and ν and the global preference weight α .
The alpha and delta functions follow the following relation:

$$\Delta = \alpha \times (\mu - \nu)$$

The Ganita Algorithm uses the preference transformation defined by $\Delta = \alpha(\mu - \nu)$ to propagate the KR condition through repeated compositions. For an intuitionistic fuzzy pair (μ, ν) satisfying $\mu > \nu$, the updated membership and non-membership values are given by

$$\mu' = \mu + \Delta, \quad \nu' = \nu - \Delta$$

If $\mu \leq \nu$, the KR condition is not satisfied and no preference adjustment is performed.

The distinguishing feature of the Ganita Algorithm is that the resulting pair (μ', ν') from one composition is used as the input for the next composition. Thus, for successive compositions,

$$(\mu_0, \nu_0) \rightarrow (\mu_1, \nu_1) \rightarrow \cdots \rightarrow (\mu_n, \nu_n)$$

where

$$\Delta_i = \alpha(\mu_i - \nu_i)$$

and

$$\mu_{i+1} = \mu_i + \Delta_i, \quad \nu_{i+1} = \nu_i - \Delta_i \quad \text{whenever} \quad \mu_i > \nu_i.$$

To illustrate this propagation, the intuitionistic fuzzy dataset $\{(0.4, 0.6), (0.5, 0.5), (0.3, 0.7), (0.6, 0.4), (0.7, 0.3)\}$ is considered with $\alpha = 0.1$ and number of preference compositions leveled at 1. The KR condition is satisfied by $(0.6, 0.4)$ and $(0.7, 0.3)$. Therefore, $(0.6, 0.4) \rightarrow (0.62, 0.38)$ and $(0.7, 0.3) \rightarrow (0.74, 0.26)$, while $(0.4, 0.6)$,

$(0.5, 0.5)$, and $(0.3, 0.7)$ remain unchanged because they do not satisfy the strict condition $\mu > \nu$.

The average changes in membership and non-membership are

$$|\Delta\mu| = 0.012, \quad |\Delta\nu| = 0.012$$

The equality follows from the fact that the same correction Δ is transferred from the non-membership degree to the membership degree.

The table 1 and the figure 1 shows the effect of α on the proposed transformation. As α increases, both the membership adjustment and expressiveness increase. This indicates that α provides a simple way to control the strength of the KR preference, with smaller values producing a more gradual adjustment.

The table 2 and the figure 2 compares the computation time of the Fuzzy, Intuitionistic Fuzzy, and preference-based Intuitionistic Fuzzy i.e KR-Intuitionistic Fuzzy models for different data sizes. The results show that the KR-based model remains computationally comparable to the other approaches and does not introduce a substantial additional computational cost for the tested data sizes.

Figure 3 compares the membership and non-membership values before and after applying the KR preference. The changes occur only for the pairs satisfying $\mu > \nu$, where membership increases and non-membership decreases. This shows that the proposed transformation strengthens an already dominant membership rather than changing every input value.

Figure 4 shows the pointwise changes produced by the KR restriction. The first three points remain unchanged, while the changes for $(0.6, 0.4)$ and $(0.7, 0.3)$ are 0.02 and 0.04, respectively. This agrees directly with $\Delta = \alpha(\mu - \nu)$ and confirms that the magnitude of the adjustment depends on the membership-non-membership gap.

Figure 5 compares computation time with the expressiveness obtained from the different models. The preference-based model i.e. KR-Intuitionistic Fuzzy based model provides expressiveness while maintaining a comparable computation time. This indicates that the proposed relation can enhance the representation of preference without imposing a significant computational burden in the considered experiment.

Alpha Sweeping Results			
Alpha (α)	Expressiveness	$\Delta\mu$	$\Delta\nu$
0.010000	0.001200	0.001200	0.001200
0.031111	0.003733	0.003733	0.003733
0.052222	0.006267	0.006267	0.006267
0.073333	0.008800	0.008800	0.008800
0.094444	0.011333	0.011333	0.011333
0.115556	0.013867	0.013867	0.013867
0.136667	0.016400	0.016400	0.016400
0.157778	0.018933	0.018933	0.018933
0.178889	0.021467	0.021467	0.021467
0.200000	0.024000	0.024000	0.024000

Table 1: Alpha Sweep Results

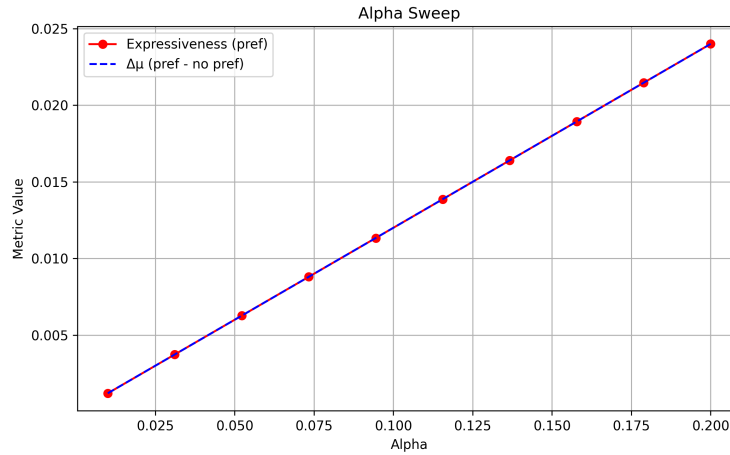


Figure 1: Alpha Sweep Graph

Efficiency Summary			
Data Size	Fuzzy Time	Intuitionistic Fuzzy Time	KR-Intuitionistic Fuzzy Time
1	0.00003719329834	0.00005602836609	0.00005006790161
5	0.00001096725464	0.00001287460327	0.00001096725464
10	0.00001215934753	0.00001406669617	0.00001120567322
15	0.00001072883606	0.00001406669617	0.00001120567322
20	0.00001215934753	0.00001406669617	0.00001120567322

Table 2: Efficiency Summary

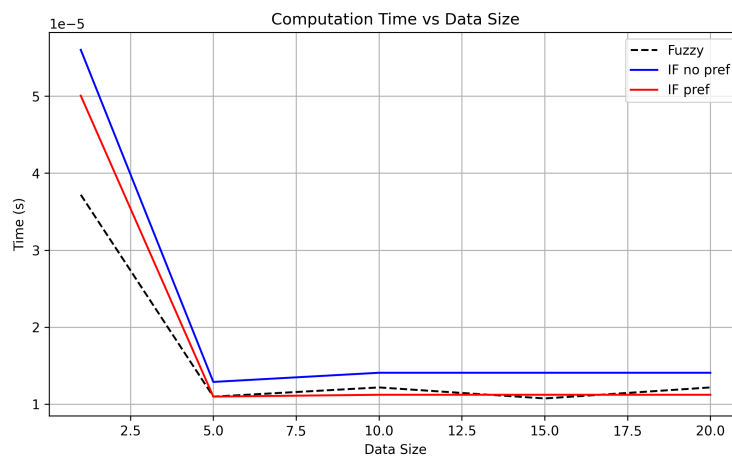


Figure 2: Efficiency Scaling

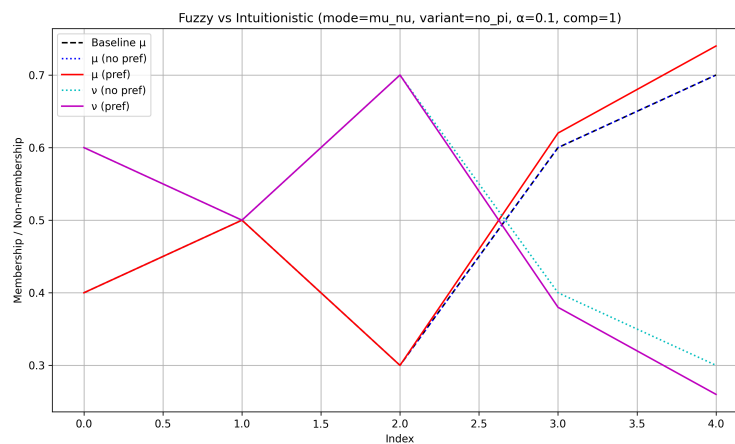
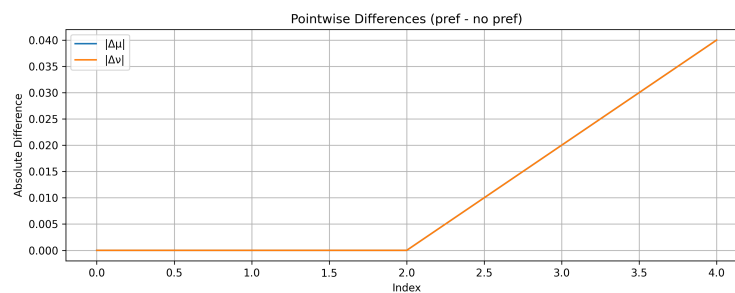


Figure 3: Membership Comparison

Figure 4: $\Delta\mu$ and $\Delta\nu$ Comparison

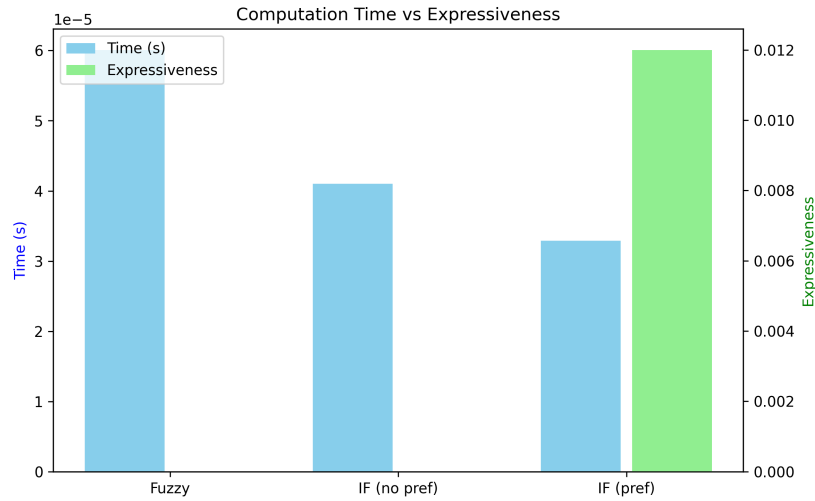


Figure 5: Computation Time and Expressiveness Comparison

Summary			
Model	Time (s)	Memory (KB)	Expressiveness Gain
Fuzzy	0.000060	0.781250	0.000
Intuitionistic Fuzzy (IF)	0.000041	0.804688	0.000
KR-Intuitionistic Fuzzy (IF-Prep)	0.000033	0.804688	0.012
Average $ \Delta\mu = 0.012000$, Average $ \Delta\nu = 0.012000$			

Overall, the results show that the KR preference selectively strengthens membership when the KR condition is satisfied, while the effect of the transformation can be controlled through α . The computational results further indicate that this additional preference mechanism can be applied without a substantial increase in computation time for the tested cases.

7. An Application: Ensuring Safety in Virtual Safety Engineer (VSE)

To demonstrate the practical applicability of the proposed KR-Intuitionistic Fuzzy Equivalence Relation, a Virtual Safety Engineer (VSE) scenario is considered. Consider an AI-based safety system in a chemical processing plant that uses Virtual Safety Engineer (VSE) to monitor hazards and process real-time data to make decisions. Its key objective is to identify and take adequate measures in case of any kind of anomalies like explosions, fire or gas leaks, using past and present data (like temperature, pressure, etc.). Now, there is a problem in this kind of

situations as a plant having multiple production units may not have identical configurations and so past incidents in any one of the unit may or may not be applicable in the other units. So, the VSE in this regard need to

- compare present state to past unsafe state,
- need to analyse the similarity between them,
- and make recommendation based on similarity and dissimilarity.

But over the time VSE's decision logic may change due to factors like new meta data, model retraining or system updates so we need to ensure that its new behaviour remains consistent with previously verified safe responses. If we consider μ : degree of alignment with safety standard and ν : degree of deviation then our KR-condition ensures that μ always remains greater than ν , thus ensuring that VSE remains consistent. How one can achieve this? Let us try to understand this.

7.1. Incident Profiles

For different parameters like Temperature (T), Pressure (P), Gas Concentration (C), Flow rate (F), Composition (S). we need to define thresholds for danger and safe zones marking it to be more dangerous as the membership values rises. VSE stores incident profiles as referencee Intuitionistic Fuzzy Vectors (μ_{I_F} , ν_{I_F}) to compute similarity and dissimilarity.

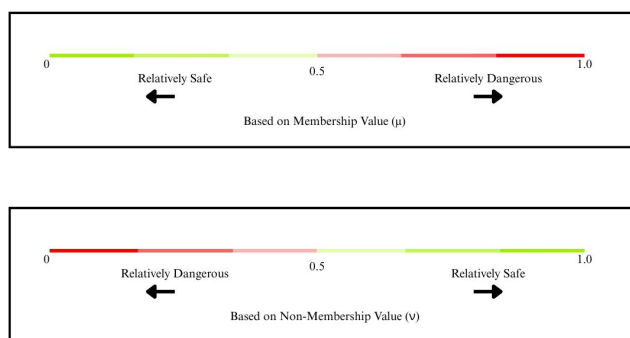


Figure 6: Zones based on Membership and Non-membership values

7.2. Parameter Profiles

The Ganita Algorithm based on the Incidents Profiles (7.1) will classify the parameters into "Priority" and "Lookout" categories, where Priority indicates higher

level of importance than Lookout. This profiling serves as a reference point and helps guide the AI by specifying what to focus on in the first place, which is especially valuable in the event of future incidents or accidents. This can be achieved with the help of a ranking system given as below

- If $\mu_i > \nu_i$ then preference will be given to the factor with the highest μ_i and will be denoted as "Priority 1".
- If $\mu_i \leq \nu_i$ then a Rank will be allotted based on Rank Value defined as $RankValue = |\mu_i - \nu_i|$ with the lowest Rank Value given the highest priority and will be denoted by "Lookout 1".

7.3. Alarm Escalation and System shutdown

Based on the results of the IFRs, if the similarity to a serious past situation is high then VSE triggers an early warning and will immediately shutdown the system and if it is moderate, it triggers a soft warning and gradually shutdown the system, and if it is low, it logs the event for human intervention and review.

7.4. Adaptive Decision-Making

The KR-condition ensures that VSE remains consistent and along with the Ganita Algorithm the VSE becomes more effective and able to analyse multiple parameters along with making multiple decisions simultaneously with the help of multi-compositional relations. This makes the system robust and effective at the same time.

7.5. An Illustration

An analysis of three systems in a Chemical Plant is presented below, detailing their respective incident profiles. These profiles are based on historical data, with percentages indicating the causal contribution of each parameter - Pressure (P), Temperature (T), Composition (S), Gas Concentration (C), and Flow Rate (F) to prior mishaps.

Incident Profile			
Factor	System 1	System 2	System 3
Pressure (P)	50%	60%	60%
Temperature (T)	70%	50%	45%
Composition (S)	60%	40%	55%
Gas Concentration (C)	30%	45%	50%
Flow Rate (F)	55%	55%	40%

Table 3: Incident Profile based on Past Events

The Intuitionistic Fuzzy Representation (IFRe) of the above data is as follows:

Fuzzification of data			
Factor	System 1	System 2	System 3
Pressure (P)	(0.50, 0.50)	(0.60, 0.40)	(0.60, 0.40)
Temperature (T)	(0.70, 0.30)	(0.50, 0.50)	(0.45, 0.55)
Composition (S)	(0.60, 0.40)	(0.40, 0.60)	(0.55, 0.45)
Gas Concentration (C)	(0.30, 0.70)	(0.45, 0.55)	(0.50, 0.50)
Flow Rate (F)	(0.55, 0.45)	(0.55, 0.45)	(0.40, 0.60)

Table 4: Intuitionistic Fuzzy Representation of Incident Profile

Based on the above Intuitionistic Fuzzy Representation, the Ganita Algorithm will generate the following Parameter Profiles assigning Priority and Lookout levels.

Parameter Profile					
System($\downarrow$)/Factor($\rightarrow$)	P	T	S	C	F
System 1	Lookout 1	Priority 1	Priority 2	Lookout 2	Priority 3
System 2	Priority 1	Lookout 1	Lookout 3	Lookout 2	Priority 2
System 3	Priority 1	Lookout 2	Priority 2	Lookout 1	Lookout 3

Table 5: Parameter Profile based on IFRe of Incident Profile

Based on the above data the VSE gives preference to Temperature in System 1 compared to the other factors, and similarly Pressure in System 2 and System 3 will be given highest preference in case of an incident.

7.6. Fail Safety

In some situations, the cause of an incident may differ from those observed in previous cases, - for instance, if earlier accidents in System 1 were primarily due to Temperature change, but the present issue arises from abnormal flow rate. Then in such a case, even if there is a mismatch between current parameters(factors) and those involved in past incidents - the VSE, empowered by the KR-Intuitionistic Fuzzy Set along with the help Ganita Alogrithm, can still detect and respond effectively. This is due to the fact that they are equipped with the Known-Restriction (3.1) which gives significance to membership values.

8. Conclusion

The proposed KR-Intuitionistic Fuzzy Equivalence Relations offers distinct advantages in multi-compositional computations, particularly within the domain of AI

safety. When multiple intuitionistic fuzzy relations are composed simultaneously (e.g., trust propagation, risk assessment, etc.), this constraint helps ensure that positive or reliable associations remain dominant throughout the process. Such a property is vital in AI safety applications, where cumulative uncertainty and conflicting evidence can undermine system reliability. Moreover, the Ganita Algorithm ensures less computational time and more efficiency and aligns to towards belongingness rather than non-belongingness. This helps to avoid overreacting to slightly abnormal but safe states, and reduce uncertainty in decisions, improving human-AI collaboration. It is not just a tool for the VSE to analyse plant hazards, but also a tool to audit and verify the AI's evolving behavior and ensure decisions stay within safety-compliant bounds. However, the proposed algorithm is not only limited to VSE but have a wide range of application in various other field including role of AI in automation including self driving cars, maps (e.g., Google Maps). One such aspect will be explored in our next potential findings related to its applicability in Fuzzy Graph.

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