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CERTAIN NEW RESULTS OF THE S-GENERALIZED GAUSS HYPERGEOMETRIC FUNCTION TRANSFORM

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Dedicated to Prof. A.M. Mathai on his 80^{th} birth anniversary

Abstract: The aim of the present paper is to further study the S-generalized Gauss hypergeometric function transform recently introduced by Srivastava, Jain and Bansal [11]. In the course of our study, we establish image of Fox H-function in the S-Generalized Gauss hypergeometric function transform and obtain the images of five useful and important cases of Fox H-function (Generalized Bessel function, Gauss Hypergeometric function, Generalized Mittag-Leffler Function, Krätzel Function and Lorenzo Hartley G-function) under the S-generalized Gauss hypergeometric function transform.

Keywords: S-Generalized Gauss hypergeometric function, Fox H-Function, Complex integral representation, Integral transform.

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1. Introduction and Definitions

S-Generalized Gauss Hypergeometric Function

The S-generalized Gauss hypergeometric function $F_p^{(\alpha,\beta;\tau,\mu)}(a,b;c;z)$ was introduced and investigated by Srivastava et al. [5, p. 350, Eq. (1.12)]. It is represented in the following manner:

$$F_p^{(\alpha,\beta;\tau,\mu)}(a,b;c;z) = \sum_{n=0}^{\infty}(a)_n \frac{B_p^{(\alpha,\beta;\tau,\mu)}(b+n,c-b)}{B(b,c-b)}\frac{z^n}{n!} \qquad (|z|<1) \tag{1.1}$$

provided that, $(\Re(p) \geq 0;\ \min\{\Re(\alpha),\Re(\beta),\Re(\tau),\Re(\mu)\} > 0;\ \Re(c) > \Re(b) > 0)$

in terms of the classical Beta function $B(\lambda, \mu)$ and the S-generalized Beta function $B_p^{(\alpha,\beta;\tau,\mu)}(x, y)$, which was also defined by Srivastava et al. [5, p.350, Eq.(1.13)] as follows:

$$B_p^{(\alpha,\beta;\tau,\mu)}(x, y) = \int_0^1 t^{x-1}(1-t)^{y-1}{}_1F_1\left(\alpha; \beta; -\frac{p}{t^\tau(1-t)^\mu}\right) dt \tag{1.2}$$

$$(\Re(p) \geq 0; \quad \min\{\Re(x), \Re(y), \Re(\alpha), \Re(\beta), \Re(\tau), \Re(\mu)\} > 0)$$

and $(\lambda)_n$ denotes the Pochhammer symbol defined (for $\lambda \in \mathbb{C}$) by (see [6, p. 2 and pp. 4-6]; see also [7, p. 2]):

$$\begin{aligned}(\lambda)_n &= \frac{\Gamma(\lambda+n)}{\Gamma(\lambda)} \\ &= \begin{cases} 1, & (n = 0) \\ \lambda(\lambda+1)...(\lambda+n-1), & (n \in \mathbb{N} \quad := \{1, 2, 3, \cdots\}) \end{cases}\end{aligned} \tag{1.3}$$

provided that the Gamma quotient exists (see, for details, [9, p. 16 et seq.] and [10, p. 22 et seq.]).

For $\tau = \mu$, the S-generalized Gauss hypergeometric function defined by (1.1) reduces to the following generalized Gauss hypergeometric function $F_p^{(\alpha,\beta;\tau)}(a, b; c; z)$ studied earlier by Parmar [14, p.44]:

$$F_p^{(\alpha,\beta;\tau)}(a, b; c; z) = \sum_{n=0}^{\infty}(a)_n \frac{B_p^{(\alpha,\beta;\tau)}(b+n, c-b)}{B(b, c-b)} \frac{z^n}{n!} \qquad (|z| < 1) \tag{1.4}$$

$$(\Re(p) \geq 0; \quad \min\{\Re(\alpha), \Re(\beta), \Re(\tau)\} > 0; \quad \Re(c) > \Re(b) > 0).$$

which, in the *further* special case when $\tau = 1$, reduces to the following extension of the generalized Gauss hypergeometric function (see, e.g., [4, p.4606, Section 3]; see also [3, p. 39]):

$$F_p^{(\alpha,\beta)}(a, b; c; z) = \sum_{n=0}^{\infty}(a)_n \frac{B_p^{(\alpha,\beta)}(b+n, c-b)}{B(b, c-b)} \frac{z^n}{n!} \qquad (|z| < 1) \tag{1.5}$$

$$(\Re(p) \geqslant 0; \quad \min\{\Re(\alpha), \Re(\beta)\} > 0; \quad \Re(c) > \Re(b) > 0)$$

Upon setting $\alpha = \beta$ in (1.5), we arrive at the following extended Gauss hypergeometric function (see [13, p.591, Eqs. (2.1) and (2.2)]:

$$F_p(a,b;c;z) = \sum_{n=0}^{\infty} (a)_n \frac{B_p(b+n, c-b)}{B(b, c-b)} \frac{z^n}{n!} \qquad (|z| < 1) \tag{1.6}$$

$$(\Re(p) \geqslant 0; \quad \Re(c) > \Re(b) > 0)$$

The S-Generalized Gauss Hypergeometric Function Transform

Srivastava and the present author have recently introduced the following S-generalized Gauss hypergeometric function transform [11]:

$$\tilde{\mathfrak{S}}[f(z); s] = \varphi(s) = \int_0^{\infty} F_p^{(\alpha,\beta;\tau,\mu)}(a,b;c;sz) f(z) dz \tag{1.7}$$

where $f(z) \in \Lambda$, and Λ denotes the class of functions for which

$$f(z) = \begin{cases} O\{z^{\zeta}\}, & (z \to 0) \\ O\{z^{w_1} e^{-w_2 z}\}, & (|z| \to \infty) \end{cases} \tag{1.8}$$

provided that the existence conditions in (1.1) for the S- generalized Gauss hypergeometric function $F_p^{(\alpha,\beta;\tau,\mu)}(.)$ are satisfied and

$$\left. \begin{array}{l} \Re(\zeta) + 1 > 0 \\ \Re(w_2) > 0 \quad \text{or} \quad \Re(w_2) = 0 \quad \text{and} \quad \Re(w_1 - a + 1) < 0 \end{array} \right\} \tag{1.9}$$

Fox H-Function

A single Mellin-Barnes contour integral, occurring in the present work, is now popularly known as the H-function of Charles Fox (1897-1977). It will be defined and represented here in the following manner (see, for example, [8, p. 10]):

$$H_{P,Q}^{M,N}[z] = H_{P,Q}^{M,N}\left[z \,\middle|\, \begin{array}{c} (a_j, \alpha_j)_{1,P} \\ \\ (b_j, \beta_j)_{1,Q} \end{array} \right] = H_{P,Q}^{M,N}\left[z \,\middle|\, \begin{array}{c} (a_1, \alpha_1), \cdots, (a_P, \alpha_P) \\ \\ (b_1, \beta_1), \cdots, (b_Q, \beta_Q) \end{array} \right]$$

$$:= \frac{1}{2\pi \mathrm{i}} \int_{\mathfrak{L}} \Theta(\mathfrak{s}) z^{\mathfrak{s}} \, d\mathfrak{s}, \tag{1.10}$$

where $\mathrm{i}=\sqrt{-1}$, $z \in \mathbb{C} \setminus \{0\}$, $\mathbb{C}$ being the set of complex numbers,

$$\Theta(\mathfrak{s}) = \frac{\prod_{j=1}^{M} \Gamma(b_j - \beta_j \mathfrak{s}) \prod_{j=1}^{N} \Gamma(1 - a_j + \alpha_j \mathfrak{s})}{\prod_{j=M+1}^{Q} \Gamma(1 - b_j + \beta_j \mathfrak{s}) \prod_{j=N+1}^{P} \Gamma(a_j - \alpha_j \mathfrak{s})}, \tag{1.11}$$

and

$$1 \leqq M \leqq Q \text{ and } 0 \leqq N \leqq P \quad (M, Q \in \mathbb{N} = \{1, 2, 3, \cdots\};\ N, P \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}), \tag{1.12}$$

an empty product being interpreted to be 1. Here $\mathfrak{L}$ is a Mellin-Barnes type contour in the complex $\mathfrak{s}$-plane with appropriate indentations in order to separate the two sets of poles of the integrand $\Theta(\mathfrak{s})$ (see, for details, [1] and [8]).

2. Main Result

In this section, we obtain the image of Fox H-function under the S - generalized Gauss hypergeometric function Transform.

2.1. The S-Generalized Gauss Hypergeometric Transform of the H-function

The S-Generalized Gauss hypergeometric Transform (1.7) of Fox H-function (1.10) defined as follows :

$$\tilde{\mathfrak{S}}\left[z^{\kappa} H_{P,Q}^{M,N} \left[Az^{\sigma} \,\middle|\, \begin{matrix} (a_j, \alpha_j)_{1,P} \\ \\ (b_j, \beta_j)_{1,Q} \end{matrix} \right]; s \right]$$

$$= \int_0^{\infty} F_p^{(\alpha,\beta;\tau,\mu)}(a, b; c; sz) z^{\kappa} H_{P,Q}^{M,N} \left[Az^{\sigma} \,\middle|\, \begin{matrix} (a_j, \alpha_j)_{1,P} \\ \\ (b_j, \beta_j)_{1,Q} \end{matrix} \right] dz$$

$$= \frac{\Gamma(\beta) A^{-\frac{\kappa+1}{\sigma}}}{\sigma \Gamma(\alpha) \Gamma(a) B(b, c-b)} H_{1,1:1+Q,1+P;3,1}^{0,1:1+N,1+M;1,2} \left[\begin{matrix} -\frac{s}{A^{\sigma}} \\ \\ \frac{1}{p} \end{matrix} \,\middle|\, \begin{matrix} (1-b; 1, \tau) : A^{*} \\ \\ (1-c; 1, \tau+\mu) : B^{*} \end{matrix} \right] \tag{2.1}$$

where
$A^{*} = (1-a, 1), (1 - b_j - \beta_j \frac{(\kappa+1)}{\sigma}, \frac{\beta_j}{\sigma})_{1,Q}; (1, 1), (1-c+b, \mu), (\beta, 1)$
$B^{*} = (0, 1), (1 - a_j - \alpha_j \frac{(\kappa+1)}{\sigma}, \frac{\alpha_j}{\sigma})_{1,P}; (\alpha, 1)$
provided that the existence conditions in (1.1) for the S-Generalized Gauss hypergeometric function $F_p^{(\alpha,\beta;\tau,\mu)}(a, b; c; z)$ are satisfied and

(i) $\min\limits_{1\leq j\leq M} \Re\left(\kappa + \frac{\sigma b_j}{\beta_j}\right) + 1 > 0$ (ii) $\max\limits_{1\leq j\leq N} \Re\left(\kappa - a + \frac{\sigma(a_j - 1)}{\alpha_j}\right) + 1 < 0$

Proof. To prove the result (2.1), we first write the complex integral representation of S-generalized Gauss hypergeometric function defined in [11] and then change the order of $\xi-$integral with z-integral (which is permissible under the conditions stated), we obtain (say Δ)

$$\Delta = \frac{1}{2\pi \mathrm{i}} \int\limits_{\mathfrak{L}} \frac{(-s)^{-u} B(u, a-u) B_p^{(\alpha,\beta;\tau,\mu)}(b-u, c-b)}{B(b, c-b)} \left\{ \int\limits_0^\infty z^{\kappa - u} H_{P,Q}^{M,N}[A z^\sigma] dz \right\} du \tag{2.2}$$

Now we evaluate the z-integral involved in (2.2) with the help of [8, p.15, Eq. (2.4.1)], we have

$$\begin{aligned} \Delta =& \frac{1}{2\pi \mathrm{i}} \int\limits_{\mathfrak{L}} \frac{(-s)^{-u} B(u, a-u) B_p^{(\alpha,\beta;\tau,\mu)}(b-u, c-b)}{B(b, c-b)} \\ & \frac{1}{\sigma} A^{-(\frac{\kappa - u + 1}{\sigma})} \frac{\prod\limits_{j=1}^{M} \Gamma(b_j + \beta_j \frac{(\kappa+1)}{\sigma} - \frac{\beta_j}{\sigma} u) \prod\limits_{j=1}^{N} \Gamma(1 - a_j - \alpha_j \frac{(\kappa+1)}{\sigma} + \frac{\alpha_j}{\sigma} u)}{\prod\limits_{j=M+1}^{Q} \Gamma(1 - b_j - \beta_j \frac{(\kappa+1)}{\sigma} + \frac{\beta_j}{\sigma} u) \prod\limits_{j=N+1}^{Q} \Gamma(a_j + \alpha_j \frac{(\kappa+1)}{\sigma} - \frac{\alpha_j}{\sigma} u)} du \end{aligned} \tag{2.3}$$

Next, we express S-generalized Beta function in terms of complex integral form. Finally, we get the right hand side of (2.1) by reinterpreting the result in terms of H-function of two variables.

Special cases of (2.1)

Here we give S-generalized Gauss hypergeometric function Transform of the some important special cases of Fox H-Function involving Generalized Bessel function, Gauss Hpergeometric function, Generalized Mittag-Leffler Function, Krätzel Function and Lorenzo Hartley G-function.

1. **S-generalized Gauss hypergeometric function Transform of Generalized Bessel Function** In (2.1), if we reduce Fox H-Function to the Generalized Bessel function [8, p.19, Eq.(2.6.10)] by taking M = 1, N = P = 0, Q = 2, $b_1 = 0, \beta_1 = 1, b_2 = -\lambda, \beta_2 = \rho$, we can easily get the following S-generalized Gauss hypergeometric function Transform of Generalized Bessel

Function after a little simplification.

$$\tilde{\mathfrak{S}}\left[z^{\kappa}J_{\lambda}^{\rho}[Az^{\sigma}];s\right]=\int_0^{\infty}F_p^{(\alpha,\beta;\tau,\mu)}(a,b;c;sz)z^{\kappa}J_{\lambda}^{\rho}[Az^{\sigma}]dz$$

$$=\frac{A^{-(\frac{\kappa+1}{\sigma})}\Gamma(\beta)}{\sigma\Gamma(\alpha)\Gamma(a)B(b,c-b)}H^{0,1:1,2;1,2}_{1,1:3,1;3,1}\left[\begin{array}{c|c}-s & (1-b;1,\tau):A^{*}\\ \frac{1}{p} & (1-c;1,\tau+\mu):B^{*}\end{array}\right] \quad (2.4)$$

where
$A^{*}=(1-a,1),(1-(\frac{\kappa+1}{\sigma}),\frac{1}{\sigma}),(1+\lambda-\rho(\frac{\kappa+1}{\sigma}),\frac{\rho}{\sigma});(1,1),(1-c+b,\mu),(\beta,1)$
$B^{*}=(0,1);(\alpha,1)$
provided that the conditions are easily obtainable from the existing conditions of (2.1) are satisfied.

2. **S-generalized Gauss hypergeometric function Transform of Gauss Hypergeometric Function** Next, if we reduce Fox H-Function to the Gauss Hypergeometric function [8, p.19, Eq.(2.6.8)] by taking M = 1, N = P = Q = 2, $a_1=1-u, a_2=1-v, b_1=0, b_2=1-w, \alpha_1=\alpha_2=\beta_1=\beta_2=1$ in (2.1), we can easily get the following S-generalized Gauss hypergeometric function Transform of Gauss Hypergeometric Function after a little simplification.

$$\tilde{\mathfrak{S}}\left[z^{\kappa}{}_2F_1[u,v;w;-Az^{\sigma}];s\right]=\int_0^{\infty}F_p^{(\alpha,\beta;\tau,\mu)}(a,b;c;sz)z^{\kappa}{}_2F_1[u,v;w;-Az^{\sigma}]dz$$

$$=\frac{A^{-(\frac{\kappa+1}{\sigma})}\Gamma(w)\Gamma(\beta)}{\sigma\Gamma(u)\Gamma(v)\Gamma(\alpha)\Gamma(a)B(b,c-b)}H^{0,1:3,2;1,2}_{1,1:3,3;3,1}\left[\begin{array}{c|c}-s & (1-b;1,\tau):A^{*}\\ \frac{1}{p} & (1-c;1,\tau+\mu):B^{*}\end{array}\right] \quad (2.5)$$

where
$A^{*}=(1-a,1),(1-(\frac{\kappa+1}{\sigma}),\frac{1}{\sigma}),(w-(\frac{\kappa+1}{\sigma}),\frac{1}{\sigma});(1,1),(1-c+b,\mu),(\beta,1)$
$B^{*}=(0,1),(u-(\frac{\kappa+1}{\sigma}),\frac{1}{\sigma}),(v-(\frac{\kappa+1}{\sigma});\frac{1}{\sigma});(\alpha,1)$
provided that the conditions are easily obtainable from the existing conditions of (2.1) are satisfied.

3. **S-generalized Gauss hypergeometric function Transform of Generalized Mittag-Leffler Function** Again, if we reduce Fox H-Function to the Generalized Mittag-Leffler function [2, p.25, Eq.(1.137)] by taking M = N = P = 1, Q = 2 and $a_1=1-\delta, b_1=0, b_2=1-\gamma, \alpha_1=\beta_1=1, \beta_2=\rho$ in (2.1), we can easily get the following S-generalized Gauss hypergeometric function

Transform of Generalized Mittag-Leffler Function after a little simplification.

$$\tilde{\mathfrak{S}}\left[z^{\kappa}E^{\delta}_{\rho,\gamma}[Az^{\sigma}];s\right]=\int_0^{\infty}F_p^{(\alpha,\beta;\tau,\mu)}(a,b;c;sz)z^{\kappa}E^{\delta}_{\rho,\gamma}[Az^{\sigma}]dz$$

$$=\frac{A^{-(\frac{\kappa+1}{\sigma})}\Gamma(\beta)}{\sigma\Gamma(\alpha)\Gamma(a)B(b,c-b)}H^{0,1:2,2;1,2}_{1,1:3,2;3,1}\left[\begin{array}{c|c} -s & (1-b;1,\tau):A^{*} \\ \frac{1}{p} & (1-c;1,\tau+\mu):B^{*}\end{array}\right] \quad (2.6)$$

where
$A^{*}=(1-a,1),(-\kappa,1),(\gamma-\rho(\kappa+1),\rho);(1,1),(1-c+b,\mu),(\beta,1)$
$B^{*}=(0,1),(\delta-\kappa-1,1);(\alpha,1)$
provided that the conditions are easily obtainable from the existing conditions of (2.1) are satisfied.

4. **S-generalized Gauss hypergeometric function Transform of Kr$\ddot{a}$tzel Function** In (2.1), if we reduce Fox H-Function to the Krätzel function [2, p.25, Eq.(1.141)] by taking M = Q = 2, N = P = 0, $b_1=0,\beta_1=1,b_2=\frac{\nu}{\rho},\beta_2=\frac{1}{\rho}$, we can easily get the following S-generalized Gauss hypergeometric function Transform of Krätzel Function after a little simplification.

$$\tilde{\mathfrak{S}}\left[z^{\kappa}Z^{\nu}_{\rho}(Az^{\sigma});s\right]=\int_0^{\infty}F_p^{(\alpha,\beta;\tau,\mu)}(a,b;c;sz)z^{\kappa}Z^{\nu}_{\rho}(Az^{\sigma})dz$$

$$=\frac{A^{-(\frac{\kappa+1}{\sigma})}\Gamma(\beta)}{\sigma\rho\Gamma(\alpha)\Gamma(a)B(b,c-b)}H^{0,1:1,3;1,2}_{1,1:3,1;3,1}\left[\begin{array}{c|c} -s & (1-b;1,\tau):A^{*} \\ \frac{1}{p} & (1-c;1,\tau+\mu):B^{*}\end{array}\right] \quad (2.7)$$

where
$A^{*}=(1-a,1),(1-(\frac{\kappa+1}{\sigma}),\frac{1}{\sigma}),(1-\frac{\nu}{\rho}-(\frac{\kappa+1}{\sigma\rho}),\frac{1}{\sigma\rho});(1,1),(1-c+b,\mu),(\beta,1)$
$B^{*}=(0,1);(\alpha,1)$
provided that the conditions are easily obtainable from the existing conditions of (2.1) are satisfied.

5. **S-generalized Gauss hypergeometric function Transform of Lorenzo Hartley G-function** In (2.1), if we reduce Fox H-Function to the Lorenzo Hartley G-function [12, p.64, Eq.(2.3)]gupta by taking M = N = P = 1, Q = 2, $a_1=1-r,\alpha_1=1,b_1=0,\beta_1=\beta_2=1,b_2=1+\nu-r$, we can easily get the following S-generalized Gauss hypergeometric function Transform of

Lorenzo Hartley G-function after a little simplification.

$$\tilde{\mathfrak{S}}\left[z^{\kappa}G_{\sigma,\nu,r}[-A,t];s\right]=\int_0^{\infty}F_p^{(\alpha,\beta;\tau,\mu)}(a,b;c;sz)z^{\kappa}G_{\sigma,\nu,r}[-A,t]dz$$
$$=\frac{A^{-(\frac{\kappa-\nu+r\sigma}{\sigma})}\Gamma(\beta)}{\sigma\Gamma(r)\Gamma(\alpha)\Gamma(a)B(b,c-b)}H^{0,1:2,2;1,2}_{1,1:3,2;3,1}\left[\begin{array}{c|c} -\frac{s}{A^{\sigma}} & (1-b;1,\tau):A^* \\ \frac{1}{p} & (1-c;1,\tau+\mu):B^* \end{array}\right] \tag{2.8}$$

where
$A^*=(1-a,1),(\frac{\sigma-\kappa+\nu-r\sigma}{\sigma},\frac{1}{\sigma}),(-\kappa,1);(1,1),(1-c+b,\mu),(\beta,1)$
$B^*=(0,1),(\frac{\nu-\kappa}{\sigma},\frac{1}{\sigma});(\alpha,1)$
provided that the conditions are easily obtainable from the existing conditions of (2.1) are satisfied.

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