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TWO PHASE FLOW OF THE CONDENSATION BOUNDARY LAYER THROUGH POROUS MEDIUM

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Abstract: In this paper, condensation heat transfer and the structure of the dispersed, two-phase, two-component boundary layer are studied under forced convection condition and through porous medium.

Keywords and Phrases: Two phase, two component flow, porous medium, boundary layer.

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1. Introduction

Hijikata and mori [1] assumed that vapour is saturated throughout the boundary layer and that droplet appear to avoid any metastable state of vapours. They neglected the heat resistance of the film, assuming that the temperature of the interface, between the liquid film and the dispersed two-phase medium was equal to the temperature of the plate of surface.

Legay [2] and Legay and Prunet [3] solved the two phase boundary layer equation, numerically by a finite difference method. Hijakata and Mori [1] pointed out that, in case of Le < 1 and for low temperature difference , a condensing vapour is in a superheated state in the single phase boundary layer. However, they did not show any numerical results to support such a statement. Further, no attention has been given to this problem in Legay [2] and Legay and Prunet[3,4] where no numerical values of droplet mass fraction were presented. At the same time, droplets were not observed in the experiment with a steam-air mixture by Legay [2] unless the temperature difference was > 20K.

These results compelled researches to deal with forced convection condensation in the presence of non-condensing gas to examine carefully the intrinsic consistency of the models based on saturated condition throughout the boundary layer. So

Matuszkiewicz and Vernier [5] , later determined the condition which should be fulfilled in order that vapour be saturated all through the boundary layer.

In this paper the main aim is to investigate the effect of porous medium of the flow and on the conditions which should be fulfilled in order that the vapour be saturated all through

Formulation Of The Problem: Consider, a homogeneous fluid mixture, consisting of non-condensing gas, saturated vapour and finely dispersed droplets of the same substance, flouring along a flat porous plate. The uniform temperature of the plate is kept lower than that of the main flow. So the vapour condenses on the plate and the liquid film arising on it is dragged plate and the liquid film arising on it is dragged by the shear stress. A laminar boundary (arises) develops in the region of the dispersed, two-phase, two component medium. In whole system only liquid film is affected by the porosity of the flat plate.

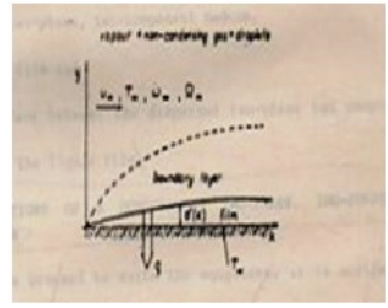

Figure 1. Physical model and coordinate system.

We propose to write the balance equations for the following:

1. Dispersed two-phase, two-component medium,

2. The liquid film and

3. The interface between the dispersed two-phase tow component medium and the liquid film.

In this case the vapour and non-condensing gas are two components in gas phase, and droplets are one of the two phases.

Thus, two different mass fractions for the problem are defined as given below:

1. The vapour mass fraction in the gas phase is denoted by w and is given by

$$w = \frac{\rho_V}{\rho_G} \tag{1}$$

where ρ_V = density of vapour and ρ_G = density of gas phase

2. The droplet mass fraction in the two phase , two component Medium is denoted by and is given by

$$\Omega = \frac{\varepsilon \rho_L}{\rho} \tag{2}$$

where ρ = density of the two-phase two-component medium, ρ_L = density of liquid phase and ε = volume fraction of the dispersed liquid phase.

The factor $\varepsilon\rho_L$ is the apparent droplet density in the two-phase, two component medium.

Equations: The equations for homogeneous two-phase, two-component boundary layer have been derived (Matuszkiewiez et al. [6].

Continuity Equation:

$$\frac{d\rho}{dt} + \rho\left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right] = 0 \tag{3}$$

Momentum Equation:

$$\rho = \frac{du}{dt} = \frac{\partial}{\partial y}\left[\mu \frac{\partial u}{\partial y}\right] \tag{4}$$

Diffusion Equation:

$$\rho(1-\Omega)\frac{dw}{dt} = \frac{\partial}{\partial y}\left[\rho(1-\Omega)D\frac{\partial w}{\partial y}\right] - \rho(1-w)\frac{d\Omega}{dt} \tag{5}$$

and

Thermal Energy Equation:

$$\rho_L \frac{dT}{dT} = \rho_L \frac{\partial \Omega}{\partial t} + \frac{\partial}{\partial y}\left[\lambda \frac{\partial T}{\partial y}\right] \tag{6}$$

where $\dfrac{d}{dt} = \dfrac{\partial}{\partial t} + u\dfrac{\partial}{\partial x} + v\dfrac{\partial}{\partial y}$, t= time, x,y = cartesian coordinates, u = velocity component parallel to the wall, v = velocity component perpendicular to the wall, μ = dynamic viscosity, D = binary diffusivity, λ = thermal conductivity, c = heat capacity, L = latent heat of evaporation, T = temperature.

Here, it is obvious that $\Omega \geq 0$ throughout the boundary layer. Equations (3) and (4) are analogous to the equations of a single-phase boundary layer. While equation (5) and (6) differ from the analogous equations of a single-phase boundary layer, as they have source terms proportional to the substantial derivative of the droplet mass fraction. In the diffusion equation, the source term is small unless the droplet mass fraction is of the same order as the vapour mass fraction. Mostly

the droplet mass function is many times less than the vapour mass fraction. So, hereafter the source term in the diffusion equation will be neglected.

Equations Of The Liquid Film: To simplify the conservation equations of the liquid film, the thin film approximations case [7] are adopted. In accordance with this approximation the inertia terms in the momentum equation and convective terms in the thermal equations are neglected. The stationary equations of the liquid film are:

Continuity:

$$\frac{\partial u_F}{\partial x} + \frac{\partial v_F}{\partial y} = 0 \tag{7}$$

Momentum:

$$\frac{\partial^2 u_F}{\partial x^2} - \frac{1}{k} u_F = 0 \tag{8}$$

and

Thermal Energy:

$$\frac{\partial^2 T_F}{\partial y^2} = 0 \tag{9}$$

where the index F refers to the film and K is the porosity of the flat plate.

Interface Balance Equations

Interface balance equation should be given along with the equation of the homogeneous, two-phase, two component boundary layer and the equations of the liquid film. In the boundary layer approximation and for a slightly curved film surface, as in our case, the interface balance equation take the following form (Matuszkieioez et al. [9].

Total mass Equation:

$$\rho \left[u \frac{d\delta}{dx} - v \right] = \rho_L \left[u_F \frac{d\delta}{dx} - v_F \right] \tag{10}$$

Momentum Equation:

$$\mu \frac{\partial u}{\partial y} = \mu_L \frac{\partial u_f}{\partial y} \tag{11}$$

Mass-Transport of non-condensing gas Equation:

$$u \frac{d\delta}{dx} - v = \frac{1}{1-w} D \frac{\partial w}{\partial y} \tag{12}$$

and

Thermal Energy Equation:

$$(1-\Omega)\rho \left[u \frac{d\delta}{dx} - v \right] L(T_j) + \lambda \frac{\partial T}{\partial y} \lambda_L \frac{\partial T_F}{\partial y} = 0 \tag{13}$$

where, δ is the film thickness and index j refers to the interface.

The interface balance equations are incomplete unless they are supported by the jump conditions, i.e. formulae relating interface. Here, continuity conditions (zero-jumps) are adopted.

In this problem, two continuity conditions are necessary. They are as follows:

(i) The continuity of tangential component of velocity. In the boundary layer approximation and for slightly curved film surface, the continuity of the tangential component of velocity is given by

$$u = u_F \tag{14}$$

(ii) The continuity of temperature. In the boundary layer approximation and for slightly curved film surface, the continuity of temperature is given by.

$$T = T_F \tag{15}$$

These two continuity conditions are equivalent to the assumption that the thermodynamic process is reversible Delhaye [11].

Relation Between Temperature And Droplet Mass Fraction

In our case, the vapours are saturated throughout the boundary. And, for saturated vapours, the vapours mass function and temperature are not independent quantities. The relation between vapour mass fraction and temperature is given by the following equation:

$$w(T) = \frac{m\rho_v(T)}{\rho - (1-m)\rho_v(T)} \tag{16}$$

where $m = \dfrac{M_v}{M_A}$, $\rho_v = \rho_v(T)$ = vapour saturation pressure, M_v= molar mass of vapour, M_A= molar mass of non-condensing gas.

If a profile of the vapours mass fraction is known then the temperature is known and thermal energy equation (6) is a first order differential equation for the droplet mass fraction.

Considering the special case of small temperature differences $(T_\infty - T_w)$ and using equation (5) and (6) the droplet mass fraction can be expressed as a function of temperature.

Firstly, we take $\Omega << w$, then the source term in the diffusion equation (5) may be dropped. In the equation (6), all the terms including gradients of the transport

coefficients are neglected. Since vapour mass fraction is a function of temperature, we may write:

$$\frac{\partial^2 T}{\partial Y^2} = \frac{1}{w_1}\frac{\partial^2 w}{\partial Y^2} - \frac{w_2}{w_1^3}\left[\frac{\partial w}{\partial y}\right]^2 \tag{17}$$

where W_1 is first temperature derivatives of w and W_2 is second temperature derivative of w.

Since, the last term $[(W_2/W_1^3)(\partial w/\partial y)^2]$ quadratic in the gradient of vapour mass fraction, so it may be neglected in the case of a small temperature difference. So, we have

$$\frac{\partial^2 T}{\partial y^2} = \frac{1}{w_1}\frac{\partial^2 w}{\partial y^2} \tag{18}$$

From the diffusion equation (5) we have the following relation:

$$\frac{1}{D}\frac{dw}{dt}\frac{\partial^2 w}{\partial y^2} \tag{19}$$

From equation (18) and (19) we have

$$\frac{\partial^2 T}{\partial y^2} = \frac{1}{dw_1}\frac{\partial w}{\partial t} \tag{20}$$

Now substituting equation (20) in the thermal energy equation (6), the following differential equation is obtained.

$$(1 - L_e)\frac{\partial T}{\partial t} = \frac{L}{C}\frac{\partial \Omega}{\partial t} \tag{21}$$

where $L_e = \dfrac{\lambda}{DPC}$.

Integration of equation (21), gives the following relation:

$$\Omega - \Omega_\infty = (1 - L_e)(C/L)(T - \Omega_\infty) \tag{22}$$

In the condensation boundary layer the temperature difference $(T - T_\infty)$ is always negative. So there are three different cases as follows:

1. For $L_e < 1$ (e.g. a steam-air mixture) the droplet mass fraction decreases as the interface is approached;

2. For $L_e > 1$ e.g. a mixture of air and ethyl alcohol vapour), the droplet mass fraction increases as the interface is approached;

3. For $L_e = 1$, the droplet mass fraction is constant.

From equation (22) it is clear that if there are no droplets can be formed in the boundary layers only for $L_e > 1$. In the case of a steam-air mixture ($L_e < 1$) the droplet mass fraction cannot be negative as the interface is approached.

The detailed analysis of thermal energy equation (6) shows that the non-linear term in the vapour mass fraction gradient, which has been neglected in equation (17) in derivation of (22), is predominant for high temperature differences (Matuszkiewies at al. [6]. Since this non-linear term is of opposite sign to the sign of other terms in the thermal energy equation. So, for $L_e < 1$, it is responsible for the raising of droplet mass fraction to positive values near the interface for $\Omega_\infty = 0$ (of. fig.4-6).

Numerical Procedure

For the complete description of the system, the equation of the condensation boundary layer must be supplemented by appropriate boundary conditions. The conditions for the main flow are obtained by fixing the vapour mass fraction and the comperature T_∞ Thus, main flow condition are:

$$u = u_\infty, T = T_\infty, w_\infty = w \circ \circ, \Omega = \Omega_\infty \tag{23}$$

The boundary conditions at the wall are:

$$u_F = 0, v_F = 0, T_F = T_w \tag{24}$$

Now, considering the stationary processes, and excluding the region near the leading edge, a similar transformation is applied to the system of condensation boundary layer quantities. It reduces the system to a system of ordinary differential equations.

Transformations for the dispersed two-phase, two-component boundary layer:

The transformations applied to the impressed two-phase, two-component boundary layer equations

$$\xi = (y - \delta)\frac{u_\infty}{v_x}, \psi = (u_\infty - v_x)^{1/2} F(\xi) \tag{25}$$

where v is kinetic viscosity.

Application of (25) to equations (3) - (6) leads to the following ordinary differential equations.

$$F''' + \frac{1}{2} F F'' = 0 \tag{26}$$

$$\Phi'' + \frac{1}{2} ScF\Phi' = 0 \tag{27}$$

and

$$\frac{L}{C}\Omega' = (1 - L_e)\frac{w_\infty - w_i}{w_1}\Phi - \left[\frac{2w_2(w_\infty - w_i)^2}{Prw_1^3}\right]\frac{\Phi'^2}{F} \tag{28}$$

where Sc is the Schmidt number , Pr is the Prandt number, and

$$\Phi = \frac{w - w_i}{w_\infty - w_i} \tag{29}$$

$$\eta = y\left[\frac{u_\infty}{v_L x}\right]^{\frac{1}{2}} \tag{30}$$

$\psi_F = (u_\infty v_L x)^{1/2} f(\eta)$;
Application of (30) to equation (7)-(9) leads to following equations:

$$f(\eta) = \frac{fw'(0)}{c}[-1 + \cosh\sqrt{C}\eta] \tag{31}$$

where $C = \dfrac{4v_L}{Ku_\infty}$ and $fw''(0)$ is the second derivative of f on the wall, and

$$\frac{T - T_w}{T_i - T_w} = \frac{\eta}{\eta_\delta} \tag{32}$$

where η_δ is the value of η for $y = \delta$.

The interface balance equations (10)-(13) are transformed with the aid of (25) and (30). Transformation of equation (10) with the aid of equation (31) the following equation is obtained

$$\delta\frac{d\delta}{dx} - \frac{1}{4}\frac{\delta^2}{x} = \frac{1}{4}\frac{\rho(vv_L)^{1/2}F(0)\eta^{2C}}{\rho_L u_\infty f''_w(0)\{-1 + \cosh\sqrt{C}\eta\}} \tag{33}$$

Solving equation (33) for δ

$$\delta(x) = \left[\frac{\rho(vv_L)^{1/2}F(0)\eta^{2C}}{\rho_L u_\infty f''_w(0)\{-1 + \cosh\sqrt{C}\eta\}}x\right]^{1/2} \tag{34}$$

Transformation of interface momentum balance equations leads to the following equation

$$Rf''_w(0) = F''(0) \tag{35}$$

where

$$R = [\rho_L v_L]/[\rho v] \tag{36}$$

Transformation of the interface balance equations of mass of non condensing gas (12) gives the following equation

$$\frac{1}{2} ScF(0) = \frac{w_\infty - w_i}{1 - w_i} \Phi'(0) \tag{37}$$

The interfacial balance equation of thermal energy (13) is transformed to the following form:

$$\frac{1}{2} F(0) \left[(1 - \Omega_i) + \frac{\lambda(1 - w_i)}{\rho D L(T_i) w_1(T_i)} \right] = \frac{\lambda_L (T_i - T_w)}{\mu_L L(T_i)} \sqrt{R} \frac{1}{\eta_\delta} \tag{38}$$

where $\eta_\delta = \left[\dfrac{2F(0)}{F''(0)} \right]^{1/2}$

Transformation of the continuity condition (14) leads to

$$F'(0) = \left[\frac{2F(0)F''(0)}{R} \right]^{1/2} \tag{39}$$

Now the main flow conditions are

$$F'(\infty) = 1, \Phi(0) = 0] \Phi(\infty) = 1, \Omega(\infty) = \Omega_\infty \tag{40}$$

The approximation [(17)(9)] involve linear profiles of velocity and temperature. Thus, it enable us to eliminate the film variables. Thus, the problem is reduced to solving the equation of the dispersed two-phase, two-component boundary layer [(26)(28)], with the aid of relation (16) non-linear boundary conditions [(37)-(40)].

This sixth order system of ordinary differential equations [(26)-(28)] has been solved by fourth order Runge-Kutta method (Matuszkiewiez et al. [9].

The model equations obtained in this work are slightly different from those obtained by Matuszkiesiez and Vernier in this paper (Matuszkiewiez) et al [8]. But the results are generally same.

Results

Experimentally calculations were made for a steam-air mixture (Le <1). Here the main stream temperature T = 373 K and

$w_1 = 0.3$

$w_2 = 0.6$

$w_3 = 0.9$
are three different values of vapour mass fraction and the droplet mass fraction, Ω is taken as 0. The range of temperature difference $(T_\infty - T_w)$ was taken between 5 and 30 K.

The observation made, are presented in the following graphs.

About Graph I: This graph is plotted for non-dimensional temperature difference between the interface and the wall against the overall temperature difference $(T_\infty - T_w)$ or three values of the main stream vapour mass fraction.

It is clear from the graph that the temperature difference between the interface and the wall does not depend strongly on the overall temperature difference but it does depend strongly on the main stream vapour mass fraction.

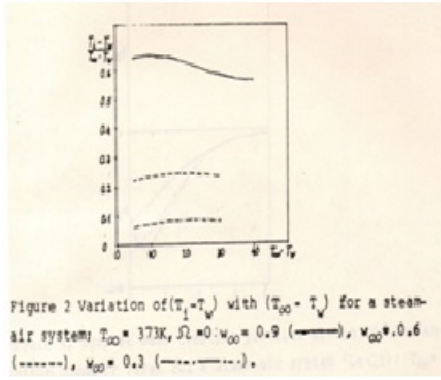

Figure 2 Variation of $(T_i - T_w)$ with $(T_\infty - T_w)$ for a steam-air system: T_∞ = 373K, Ω = 0, w_∞ = 0.9 (———), w_∞ = 0.6 (------), w_∞ = 0.3 (—·—·—·—).

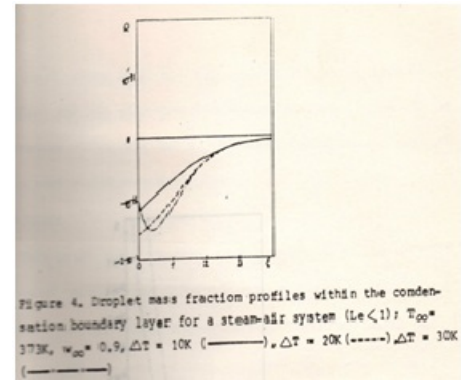

Figure 4. Droplet mass fraction profiles within the condensation boundary layer for a steam-air system (Le < 1): T_∞ = 373K, w_∞ = 0.9, ΔT = 10K (———), ΔT = 20K (-----), ΔT = 30K (—·—·—·—)

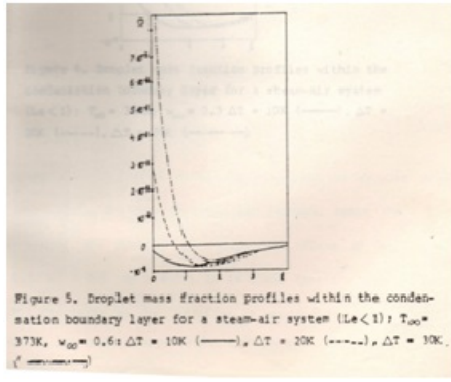

Figure 5. Droplet mass fraction profiles within the condensation boundary layer for a steam-air system (Le < 1): T_∞ = 373K, w_∞ = 0.6; ΔT = 10K (———), ΔT = 20K (-----), ΔT = 30K (—·—·—·—)

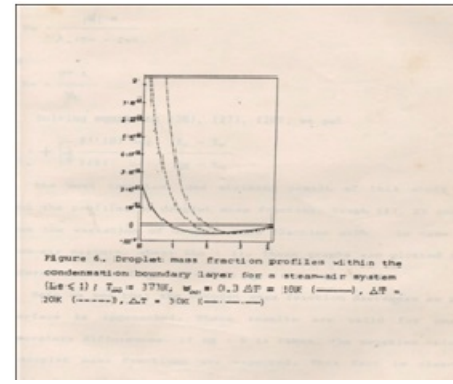

Figure 6. Droplet mass fraction profiles within the condensation boundary layer for a steam-air system (Le < 1): T_∞ = 373K, w_∞ = 0.3 ΔT = 10K (———), ΔT = 20K (-----), ΔT = 30K (—·—·—·—)

About Graph II

The heat flux transferred to the wall is shown in this graph. The results of

calculations are expressed in terms of the quantity $Nu/Re(Re)^{1/2}$, where

$$N_u = \frac{|d|}{\lambda_L(T_\infty - T_w)}$$

and

$$Re\frac{u_\infty x}{v_L}$$

Solving equations (26), (27), (28), we get

$$\frac{N_u}{Re} = + \left[\frac{1}{2}\frac{F''(0)}{F(0)}\right]^{1/2} \frac{T_i - T_w}{T_\infty - T_w}$$

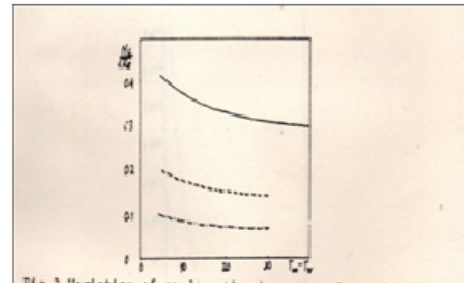

Fig. 3 Variation of condensation heat transfer with temperature difference for a steam-air system; T_∞= 373K, Ω_∞=0 ; w_∞ = 0.9 (———), w_∞=0.6 (-------), w_∞ = 0.3(- . . . .)

The most important and striking result of this study is about the profiles of droplet mass fraction. Graph III, IV and V shows the variation of droplet mass fraction with in case of steam-air mixture, where ($L_e < 1$). These graphs are plotted for different values of $w_\infty (= 0.3, w_\infty = 0.6, w_\infty = 0.9)$

Here we see that the droplet mass fraction decreases as the interface is approached. These results are valid for small temperature differences. If Ω_∞ is taken, the negative values of droplet mass fractions are expected. This fact is clearly visible in graphs III, IV and V.

All though negative values of droplet mass fraction are physically meaningless, it may be concluded that the vapours is superheated in this region.

Hence, the saturation condition is not to be imposed throughout the boundary layer for $L_e < 1$, when there are no droplets in the main flow i.e. $\Omega_\infty = 0$. Thus for $L_e > 1$ the, condition $\Omega \geq 0$, should be satisfied all along the boundary layer. While, for $L_e < 1$, it is satisfied only for the following condition i.e.

$$\Omega_\infty > \Omega_\infty \min$$

where

$$\Omega_\infty \min = (1 - L_e)\frac{C}{L}\left[\frac{T_\infty - T_i}{L(T_\infty)}\right]$$

Also, from the model equations derived we see that there is a considerable change in the film thickness equation.

Conclusions: From the results we can say that the main conclusions of this paper are:

1. As the temperature decreases in the condition boundary layer the droplet mass fraction decreases slightly in an external part of the boundary. And droplet mass fraction increases rapidly near the interface for Le<1.

2. The heat flux transferred to the wall is only slightly influenced by droplets present in the condensation boundary layer.

3. The assumption that $T_i = T_w$, is not acceptable unless the vapour mass fraction is very low , e.g. < 0.1 for the steam-air mixture.

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