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AN INNOVATIVE APPROACH ON NANO SOFT TOPOLOGICAL SPACE

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Abstract: In this paper, we investigate a new method of nano topological space stimulated by soft set. The soft set is generated in lower approximation, upper approximation and boundary region, called as **"Nano Soft Topological Space(NSTS)"**. We study the properties of soft approximation space $(\mathcal{U}, F_A)$ in NSTS. Further, we define with an example a new soft matrices induced by binary relation through the NSTS. To study the real life application, we carried out an comparative analysis between nano topological space and NSTS induced by soft matrix.

Keywords and Phrases: Soft lower approximation, Soft upper approximation, Soft boundary region, Soft approximation space, Soft matrix.

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1. Introduction

Lellis Thivagar et.al [3] introduced a nano topological space with respect to a subset X of an universe, which is defined in terms of lower and upper approximations and boundary region. The nano topological space is discussed in approximation space$(\mathcal{U}, R)$. The concept of soft set was first introduced by Molodtsov [5] in 1999 as a general mathematical tool for studying the uncertain objects. In this paper we define nano soft topological space using soft set. We also study the soft approximation space $(\mathcal{U}, F_A)$ in nano soft topological space. The properties of soft lower and soft upper approximations are discussed with an example. Further, we defined and studied the properties of soft matrix based on nano soft topological space. An example containing a comparative analysis between approximation space based on nano topological space [3] and nano soft topological space induced

by soft matrix are also discussed.

2. Preliminaries

Definition 2.1: [5,6] A soft set (F, A) denoted by F_A on the universe $\mathcal{U}$ is defined by the set of ordered pairs $F_A = \{(e, F(e)) : e \in E, F(e) \in P(\mathcal{U})\}$, where $F : E \to P(\mathcal{U})$ such that $F(e) = \emptyset$ if $e \notin A$. Here, F is called an approximate function of the soft set F_A.The set F(e) is called e-approximate value set or e-approximate set which consists of related objects of the parameter $e \in E$.

Definition 2.2: [5] A soft set (F, A) over $\mathcal{U}$ is said to be NULL soft set denoted by $\tilde{\emptyset}$, if $\forall e \in A$,F(e)=$\tilde{\emptyset}$.

Definition 2.3: [5] A soft set (F, A) over $\mathcal{U}$ is said to be an absolute soft set denoted by $\tilde{\mathcal{U}}$, if $\forall e \in A$,F(e)=$\mathcal{U}$.

Definition 2.4: [5] A union of two soft sets (F, A) and (G, B) over a common universe $\mathcal{U}$ is the soft set (H, C),where $C = A \cup B$,and $\forall e \in C$,

$$H(e) = \begin{cases} F(e) & \text{if } e \in A - B \\ G(e) & \text{if } e \in B - A \\ F(e) \cup G(e) & \text{if } e \in A \cap B \end{cases}$$

We write $(F, A)\tilde{\cup}(G, B) = (H, C)$.

Definition 2.5: [5] An intersection of two soft sets (F, A) and (G, B) over a common universe $\mathcal{U}$ is the soft set (H, C),where $C = A \cap B$,and $\forall e \in C, H(e) = F(e) \cap G(e)$. We write $(F, A)\tilde{\cap}(G, B) = (H, C)$.

Definition 2.6: [5] For two soft sets (F, A) and (G, B) over a common universe $\mathcal{U}$, we say that (F, A) is a soft subset of (G, B) if

(i) $A \subset B$ and

(ii) $\forall e \in A, F(e)$ and $G(e)$ are identical approximations. we write $(F, A)\tilde{\subset}(G, B)$. (F, A) is said to be a soft super set of (G, B), if (G, B) is a soft subset of (F, A).We denote it by $(F, A)\tilde{\supset}(G, B)$.

Definition 2.7: [1] Let (F, A) and (G, B) be two soft sets over $\mathcal{U}$,then the Cartesian product of (F,A) and (G,B) is defined as,$(F, A) \times (G, B) = (H, A \times B)$,where $H : A \times B \to P(\mathcal{U} \times \mathcal{U})$ and $H(a, b) = F(a) \times G(b)$,where $\forall (a, b) \in A \times B$,i.e., $H(a, b) = \{(h_i, h_j) : h_i \in F(a) \text{ and } h_j \in G(b)\}$.

Definition 2.8: [1] Let F_A and G_B be two soft sets over $\mathcal{U}$, then a soft binary

relation from F_A to G_B is a soft subset of $F_A \times G_B$.

In other words, a soft binary relation from F_A to G_B is of the form H'_C,where $C \subseteq A \times B$ and $H'(a,b) = H(a,b), \forall (a,b) \in C$,and $H_{A\times B} = F_A \times G_B$ as defined in Definition 2.7.Any Subset of $F_A \times F_A$ is called a soft binary relation on F_A.

In an equivalent way, we can define the soft binary relation R on the soft set F_A in the parameterized form as follows: If $F_A = \{F(a), F(b), ...\}, a, b, ... \in A$,then $F(a)RF(b)$ iff $F(a) \times F(b) \in R$.

Definition 2.9: [12] Let R be a soft equivalence relation on F_A,then

(i) soft reflexive if $F(a) \times F(a) \in R, \forall a \in A$.

(ii) soft symmetric if $F(a) \times F(b) \in R \Rightarrow F(b) \times F(a) \in R, \forall a, b \in A$.

(iii) soft transitive if $F(a) \times F(b) \in R, F(b) \times F(c) \in R \Rightarrow F(a) \times F(c) \in R, \forall a, b, c \in A$.

Definition 2.10: [1,12] Let F_A be a soft set, then soft equivalence class of $F(a)$ denoted by $[F(a)]$ is defined as $[F(a)] = \{F(b) : F(a) \times F(b) \in R, \forall a, b \in A\}$.

Remark 2.11: [1,12] For $A \subseteq E$ and F_A be a soft set and A_a denote all those elements in A corresponding to $[F(a)]$, it can be seen that $[F(a)] = (F, A_a), a \in A$ is a soft subset of F_A,where $A_a = \{b \in A : F(a) \times F(b) \in R\}$.

Definition 2.12: [1,12] Let $\mathcal{U}$ be the non empty finite universe and (F_A, R) is a soft approximation space, where F_A is a soft set over $\mathcal{U}$ and R is a soft equivalence relation on F_A.

Definition 2.13: [8] Let $\tilde{\tau}$ be the collection of soft sets over $\mathcal{U}$, then $\tilde{\tau}$ is called a soft topology on $\mathcal{U}$, if $\tilde{\tau}$ satisfies the following axioms:

(i) $\tilde{\emptyset}, \tilde{\mathcal{U}} \in \tilde{\tau}$.

(ii) The union of any number of soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$.

(iii) The intersection of any two soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$.

The triplet $(\mathcal{U}, \tilde{\tau}, E)$ is called a soft topological space over $\mathcal{U}$, then the members of $\tilde{\tau}$ are said to be soft open sets in $\mathcal{U}$.

Definition 2.14: [3] Let $\mathcal{U}$ be a non empty finite set of objects called the universe, R be an equivalence relation on $\mathcal{U}$ named as the indiscerniblity relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair $(\mathcal{U}, R)$ is said to be approximation space. Let $X \subseteq \mathcal{U}$.

(i) The Lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \left\{ \bigcup_{x \in \mathcal{U}} \{R(x) : R(x) \subseteq X\} \right\}$, where R(x) denotes the equivalence class determined by x.

(ii) The Upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X) = \left\{ \bigcup_{X \in \mathcal{U}} \{R(x) : R(x) \cap X \neq \emptyset\} \right\}$.

(iii) The Boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as not X with respect to R and it is denoted by $B_R(X) = U_R(X) - L_R(X)$.

Definition 2.15: [3] Let $\mathcal{U}$ be the universe,R be an equivalence relation on $\mathcal{U}$ and $\tau_R(X) = \{\mathcal{U}, \emptyset, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq \mathcal{U}$ and $\tau_R(X)$satisfies the following axioms.

(i) $\mathcal{U}$ and $\emptyset \in \tau_R(X)$.

(ii) The union of the elements of any sub collection$\tau_R(X)$ is in $\tau_R(X)$.

(iii) The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ forms a topology on $\mathcal{U}$ and its called as the nano topology on $\mathcal{U}$ with respect to X. Throughout this paper a binary relation is called as a relation.

3. Soft Approximation Space Based On Nano Topological Space

In this section, we define the nano topological space induced by soft set is said to be "nano soft topological space" and their properties are investigated.

Definition 3.1: Let $\mathcal{U}$ be non empty finite universe, F_A be a soft set over $\mathcal{U}$ and R is a soft equivalence relation on F_A. Elements belonging to the soft equivalence class of $F(a)$ denoted by $[F(a)]$ are said to be soft indiscernible with one another. The ordered pair $(\mathcal{U}, F_A)$ is said to be soft approximation space. Let $G_B \subseteq F_A$

(i) If $L_R(G_B) = \bigcup_{a \in A} \{F(a) : [F(a)] \subseteq G_B\}$ is a soft lower approximation of F_A with respect to G_B.

(ii) If $U_R(G_B) = \bigcup_{a \in A} \{F(a) : [F(a)] \cap G_B \neq \tilde{\emptyset}\}$ is a soft upper approximation of F_A with respect to G_B.

(iii) If $B_R(G_B) = U_R(G_B) - L_R(G_B)$ is a boundary region of F_A with respect to G_B.

Definition 3.2: Let $\mathcal{U}$ be a non empty finite set of objects called the universe, F_A is an soft set over $\mathcal{U}$. Then $(\mathcal{U}, F_A)$ is an ordered pair of soft approximation space and $\tilde{\tau}_R(G_B) = \{\tilde{\mathcal{U}}, \tilde{\emptyset}, L_R(G_B), U_R(G_B), B_R(G_B)\})$, where $G_B \subseteq F_A$, and $\tilde{\tau}_R(G_B)$ satisfies the following axiom:

(i) $\tilde{\mathcal{U}}, \tilde{\emptyset} \in \tilde{\tau}_R(G_B)$.

(ii) The union of the elements of any sub collection of soft sets $\tilde{\tau}_R(G_B)$ is an $\tilde{\tau}_R(G_B)$.

(iii) The intersection of the elements of any finite sub collection of soft sets $\tilde{\tau}_R(G_B)$ is an $\tilde{\tau}_R(G_B)$.

That is, $\tilde{\tau}_R(G_B)$ forms a soft topology on $\mathcal{U}$ having the at most five elements of soft sets and triplet ordered pair of $(\mathcal{U}, \tilde{\tau}_R, E)$ is called a nano soft topological space over $\mathcal{U}$ with respect to G_B, then the members of $\tilde{\tau}_R$ are said to be nano soft open sets in $\mathcal{U}$.

Remark 3.3: From the above definition it is clear that $L_R(G_B), U_R(G_B)$ and $B_R(G_B)$ are soft subsets of F_A.

Example 3.4: Let $\mathcal{U} = \{h_1, h_2, h_3, h_4, h_5\}$ be the universe, $E = \{a_1, a_2, a_3, a_4, a_5\}$ and $A = \{a_1, a_2, a_3, a_4\}$ be a set of parameters. Let $F_A = \{(a_1, \{h_1, h_2\}), (a_2, \{h_2, h_3\}), (a_3, \{h_2, h_3\}), (a_4, \{h_4, h_5\})\}$ and $G_B \subseteq F_A$, where $G_B = \{(a_1, \{h_1, h_2\}), (a_2, \{h_3\}, (a_3, \{h_2, h_3\})$ such that $F(a_1) = \{h_1, h_2\}$, $F(a_2) = \{h_2, h_3\}$, $F(a_3) = \{h_2, h_3\}$, $F(a_4) = \{h_4, h_5\}$ is a soft set over $\mathcal{U}$ and $R = \{F(a_1) \times F(a_1), F(a_2) \times F(a_2), F(a_3) \times F(a_3), F(a_4) \times F(a_4), F(a_1) \times F(a_2), F(a_2) \times F(a_1)\}$ is a soft equivalence relation. Then $[F(a_1)] = \{F(a_1), F(a_2)\}$, $[F(a_2)] = \{F(a_1), F(a_2)\}$, $[F(a_3)] = \{F(a_3)\}, [F(a_4)] = \{F(a_4)\}$. We have $L_R(G_B) = \{(a_3, \{h_2, h_3\})\}$ and $U_R(G_B) = \{(a_1, \{h_1, h_2\}), (a_2, \{h_2, h_3\}), (a_3, \{h_2, h_3\})\}$ and $B_R(G_B) = \{(a_1, \{h_1, h_2\}), (a_2, \{h_2, h_3\})\}$. Hence $\tilde{\tau}_R(X) = \{\tilde{\mathcal{U}}, \tilde{\emptyset}, \{(a_3, \{h_2, h_3\})\}, \{(a_1, \{h_1, h_2\}), (a_2, \{h_2, h_3\}), (a_3, \{h_2, h_3\})\}, \{(a_1, \{h_1, h_2\}), (a_2, \{h_2, h_3\})\}\}$ is a nano soft topological space.

Definition 3.5: Let F_A be a soft set over $\mathcal{U}$, and $(\mathcal{U}, F_A)$ be a soft approximation space and $G_B \subseteq F_A$. We define the characterization of five basic types of nano soft topological space as follows as:

(i) If $L_R(G_B) = \tilde{\emptyset}$ and $U_R(G_B) = \tilde{\mathcal{U}}$, then $\tilde{\tau}_R(G_B) = \{\tilde{\mathcal{U}}, \tilde{\emptyset}\}$ is called as nano indiscrete soft topology on $\mathcal{U}$.

(ii) If $L_R(G_B) = U_R(G_B) = \tilde{\mathcal{U}}$, then the roughly nano soft topology ,$\tilde{\tau}_R(G_B) = \{\tilde{\mathcal{U}}, \tilde{\emptyset}, L_R(G_B)\}$.

(iii) If $L_R(G_B) = \tilde{\emptyset}$ and $U_R(G_B) \neq \tilde{\mathcal{U}}$, then internally nano soft topology ,$\tilde{\tau}_R(G_B) = \{\tilde{\mathcal{U}}, \tilde{\emptyset}, U_R(G_B)\}$.

(iv) If $L_R(G_B) \neq \tilde{\emptyset}$ and $U_R(G_B) = \tilde{\mathcal{U}}$, then externally nano soft topology $\tilde{\tau}_R(G_B) = \{\tilde{\mathcal{U}}, \tilde{\emptyset}, L_R(G_B), B_R(G_B)\}$.

(v) If $L_R(G_B) \neq U_R(G_B)$, where $L_R(G_B) \neq \tilde{\emptyset}$ and $U_R(G_B) \neq \tilde{\mathcal{U}}$, then nano discrete soft topology on $\tilde{\tau}_R(G_B) = \{\tilde{\mathcal{U}}, \tilde{\emptyset}, L_R(G_B), U_R(G_B), B_R(G_B)\}$.

Proposition 3.6: Let $(\mathcal{U}, \tilde{\tau_R}, E)$ be a nano soft topological space with respect to G_B. Let $G_B, H_C \subseteq F_A$. Then

(i) $L_R(\tilde{\emptyset}) = U_R(\tilde{\emptyset}) = \tilde{\emptyset}$.

(ii) $L_R(\tilde{\mathcal{U}}) = \tilde{\mathcal{U}}$ and $U_R(\tilde{\mathcal{U}}) = \tilde{\mathcal{U}}$.

(iii) If $G_B \subseteq H_C$ then $L_R(G_B) \subseteq L_R(H_C)$ and $U_R(G_B) \subseteq U_R(H_C)$

(iv) $L_R(G_B) = (U_R(G_B^C))^C$ and $U_R(G_B) = (L_R(G_B^C))^C$.

(v) $L_R(G_B \cap H_C) = L_R(G_B) \cap L_R(H_C)$ and $L_R(G_B) \cup L_R(H_C) \subseteq L_R(G_B \cup H_C)$.

(vi) $U_R(G_B \cup H_C) = U_R(G_B) \cup U_R(H_C)$ and $U_R(G_B) \cap U_R(H_C) \supseteq U_R(G_B \cap H_C)$.

Proof:

(i) We can easily obtain that,by definition we get $L_R(\tilde{\emptyset}) = \bigcup_{a \in A} \{F(a) : [F(a)] \subseteq \tilde{\emptyset}\} = \tilde{\emptyset}$,and similarly we have $U_R(\tilde{\emptyset}) = \bigcup_{a \in A} \{F(a) : [F(a)] \cap \tilde{\emptyset} \neq \tilde{\emptyset}\}$. Thus we have $L_R(\tilde{\emptyset}) = \tilde{\emptyset} = U_R(\tilde{\emptyset})$. Hence we have proved.

(ii) Similarly, we can prove that the proof (i).

(iii) Let $G_B \subseteq H_C$ and $F(a) \in L_R(G_B), a \in A$, then by definition $[F(a)] \subseteq G_B$, and so $[F(a)] \subseteq H_C$.Thus $F(a) \in L_R(H_C)$ and we can say that $L_R(G_B) \subseteq L_R(H_C)$. Similarly, we can show that $U_R(G_B) \subseteq U_R(H_C)$.

(iv) We have $F(a) \in (U_R(G_B)^C)^C$ iff $F(a) \notin (U_R(G_B)^C)^C$ iff $[F(a)] \cap G_B^C = \emptyset$, then by definition, we can get $[F(a)] \subseteq G_B$ iff $F(a) \in G_B$. Thus $L_R(G_B) = (U_R(G_B)^C)^C$. Similarly, we can prove that $U_R(G_B) = (L_R(G_B)^C)^C$.

(v) For all $a \in A$, $F(a) \in L_R(G_B \cap H_C)$, we have $[F(a)] \in L_R(G_B \cap H_C)$. Since we can obtain that $[F(a)] \subseteq G_B \cap H_C$. Now by definition we can get $[F(a)] \subseteq G_B$ and $[F(a)] \subseteq H_C$.Then we can say that $F(a) \in L_R(G_B)$ and $F(a) \in L_R(H_C)$, which implies that $F(a) \in L_R(G_B \cap H_C)$. Thus, we can get $L_R(G_B \cap H_C) \subseteq L_R(G_B) \cap L_R(H_C)$. Similarly $L_R(G_B \cap H_C) \supseteq L_R(G_B) \cap L_R(H_C)$.Hence $L_R(G_B \cap H_C) = L_R(G_B) \cap L_R(H_C)$. Similarly, we can prove that $L_R(G_B \cup H_C) \supseteq L_R(G_B) \cup L_R(H_C)$.

(vi) Similarly, it follows from the proof (v).

Proposition 3.7: If $(\mathcal{U}, \tilde{\tau_R}, E)$ is a nano soft topological space. Then for all $G_B \subseteq F_A$

(i) $L_R(G_B) \subseteq G_B$ and $U_R(G_B) \supseteq G_B$.

(ii) $U_R(L_R)(G_B) \subseteq L_R(G_B)$ and $L_R(U_R)(G_B) \supseteq U_R(G_B)$.

(iii) $U_R(U_R)(G_B) \subseteq U_R(G_B)$ and $L_R(L_R)(G_B) \supseteq L_R(G_B)$.

4. Soft Matrix Based on Nano Soft Topology

In this section we will give the basic deviations for soft matrix induced by binary relation to define the nano soft topological space. Through out this section binary relation represent as relation.

Definition 4.1: Let $\mathcal{U}$ be the universe and E be set of parameters, where $A \subseteq E$, then the ordered pair $(\mathcal{U}, F_A)$ is an soft approximation space. For an binary relation R over $\mathcal{U} \times E$ .We can define function $f_A : A \longrightarrow P(\mathcal{U})$ by $R_s = \{(u, x) : x \in A, u \in f_A(x)\}$ and $R_s(x) = \{u|(u, x) \in R_s\} = R_s$ is a binary relation on soft set over $\mathcal{U}$, then the characteristic function of α_{R_S} is written by

$$\alpha_{R_S}(u, x) = \begin{cases} 1 & \text{if } (u, x) \in R_S \\ 0 & \text{if } (u, x) \notin R_S \end{cases}$$

If $[a_{ij}] = \alpha_{R_S}(u_i, x_j)$,where i=1,2,...n and j=1,2,....m. Now,we can define relation based on soft matrix as follows.

$$[a_{ij}] = \begin{pmatrix} a_{11} & a_{12}..... & a_{1n} \\ a_{21} & a_{22}..... & a_{2n} \\ . & . & . \\ . & . & . \\ . & . & . \\ a_{m1} & a_{m2}..... & a_{mn} \end{pmatrix}$$

which is called as $m \times n$ relation in soft matrix of the soft set F_A over $\mathcal{U}$ and its denoted by $SM_{m\times n}$.

Definition 4.2: Let F_A be a soft set over $\mathcal{U}$, where $\mathcal{U}$ is an universal set and $(\mathcal{U}, F_A, R_s)$ is an triple ordered pair of soft approximation space induced by relation R_s. For any $H_B \subseteq F_A$, then we call lower approximation of soft matrix, upper approximation of soft matrix and boundary region of soft matrix of H_B with respect to F_A are denoted by $L_{R_s}(H_B), U_{R_s}(H_B)$ and $B_{R_s}(H_B)$ are defined respectively as follows:

(i) $L_{R_s}(H_B) = \bigcup_{x\in A} \{R_s(x) | R_s(x) \subseteq H_B\}$.

(ii) $U_{R_s}(H_B) = \bigcup_{x\in A} \{R_s(x) | R_s(x) \cap H_B \neq \emptyset\}$.

(iii) $B_{R_s}(H_B) = U_{R_s}(H_B) - L_{R_s}(H_B)$.

That is, $\tilde{\tau}_{R_s}(H_B) = \{\tilde{\mathcal{U}}, \tilde{\emptyset}, L_{R_s}(H_B), U_{R_s}(H_B), B_{R_s}(H_B)\}$ forms a soft topology on $\mathcal{U}$ called the nano soft topology induced by soft matrix on $\mathcal{U}$ with respect to F_A. We call $(\mathcal{U}, \tilde{\tau}_{R_s}, E)$ as the nano soft topological space induced by soft matrix.

Example 4.3: Consider $\mathcal{U} = \{u_1, u_2, u_3, u_4, u_5\}$ and $E = \{a_1, a_2, a_3, a_4\}$, $A = \{a_1, a_2, a_3\}$ be a parameter and $F_A = \{(a_1, \{u_1, u_3, u_5\}), (a_2, \{u_2, u_3\}), (a_3, \{u_4\})\}$ and $H_B = \{(a_1, \{u_1, u_3, u_5\}), (a_2, \{u_2\}), (a_3, \{u_4\})\}$ be a soft set over $\mathcal{U}$. Then arbitrary relation R_s by R_s=$\{(u_1, a_1), (u_2, a_2), (u_3, a_1), (u_3, a_2), (u_4, a_3), (u_5, a_1)\}$, where $R_s(a_1) = \{u_1, u_3, u_5\} = (a_1, \{u_1, u_3, u_5\})$, $R_s(a_2) = \{u_2, u_3\} = (a_2, \{u_2, u_3\}), R_s(a_3) = \{u_4\} = (a_3, \{u_4\})$ is an binary relation on soft set. Then soft matrix is

$$[a_{ij}] = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

Then $\tilde{\tau}_{R_s}(H_B) = \{\tilde{\mathcal{U}}, \tilde{\emptyset}, \{(a_1, \{u_1, u_3, u_5\}), (a_3, \{u_4\})\}, \{(a_2, \{u_2, u_3\})\}\}$. Hence $\tilde{\tau}_{R_s}(H_B)$ is an soft matrix based on nano soft topology.

Proposition 4.4: Let $(\mathcal{U}, F_A, R_s)$ be an soft approximation space induced by any relation and $H_B \subseteq F_A$. We say that:

(i) A soft matrix of order $m \times n$ is said to be null soft matrix based on nano soft topology,if all of its elements are zero, denoted by $\emptyset$. If $L_{R_s}(H_B) = \tilde{\emptyset}$,$U_{R_s}(H_B) = \tilde{\emptyset}$ and $\tilde{\tau}_{R_S}(H_B) = \{\tilde{\mathcal{U}}, \tilde{\emptyset}\}$.

(ii) A soft matrix of order $m \times 1$ ie.,with a single column is called a column soft matrix based on nano soft topological space. Single column formally corresponds to a soft set whose parameter set contains only one parameter. If $L_{R_s}(H_B) \neq \tilde{\emptyset}$ and $\tilde{\tau}_{R_s}(H_B) = \{\tilde{\mathcal{U}}, \tilde{\emptyset}, L_{R_s}(H_B)\}$ also $U_{R_s}(H_B) \neq \tilde{\emptyset}$, and $\tilde{\tau}_{R_s}(H_B) = \{\tilde{\mathcal{U}}, \tilde{\emptyset}, U_{R_s}(H_B)\}$.

(iii) A soft matrix of order $1 \times n$ ie., with a single row is called a row soft matrix. Single row soft matrix formally corresponds to a soft set whose universal set only one object. If $H_B \subseteq F_A$ and $L_{R_s}(H_B) = \tilde{\mathcal{U}}$ and $\tilde{\tau}_{R_s}(H_B) = \{\tilde{\mathcal{U}}, \tilde{\emptyset}\}$.

(iv) A soft matrix of order $m \times n$ is said to be square soft matrix. If m = n ie.,the number of rows and the number of columns are equal. Square soft matrix is formally equal to a soft set having same number of object and parameter. If soft matrix induced all possible nano soft topology (ii)& (iii) & (iv) are available ie.,$H_B \subseteq F_A$ and $L_{R_s}(H_B) \neq \tilde{\emptyset}, U_{R_s}(H_B) \neq \tilde{\emptyset}$ and $\tilde{\tau}_{R_s}(H_B) = \{\tilde{\mathcal{U}}, \tilde{\emptyset}, L_{R_s}(H_B), U_{R_s}(H_B), B_{R_s}(H_B)\}$.

(v) A soft matrix of order $m \times n$ is said to be a complete soft matrix, if all of its elements are one. If $L_{R_s}(H_B) = \tilde{\mathcal{U}}, U_{R_S}(H_B) = \tilde{\mathcal{U}}$ and $\tilde{\tau}_{R_s}(H_B) = \{\tilde{\mathcal{U}}, \tilde{\emptyset}\}$.

(vi) A soft matrix of order $m \times n$ is said to be a diagonal soft matrix,if all of its non-diagonal elements are zero. If $L_{R_s}(H_B) = U_{R_s}(H_B)$and $\tilde{\tau}_{R_s}(H_B) = \{\tilde{\mathcal{U}}, \tilde{\emptyset}, L_{R_s}(H_B)\}$.

Example 4.5: Let $\mathcal{U} = \{d_1, d_2, d_3, d_4, d_5\}$ be the set of five dresses and the set of parameter $E = \{costly, beautiful, cheap, comfortable, gorgeous\} = \{e_1, e_2, e_3, e_4, e_5\}$. Let $(\mathcal{U}, F_A, R_s)$ be an soft approximation space induced by any relation and $H_B \subseteq F_A$.Then

(i) Let $\mathcal{U} = \{d_1, d_2, d_3, d_4\}$ be the four dresses and $E = \{costly, beautiful, cheap, comfortable, gorgeous\} = \{e_1, e_2, e_3, e_4, e_5\}$. Let A = $\{e_2, e_3, e_4, e_5\} \subseteq E$ and $F_A = \{(e_2, \emptyset), (e_3, \emptyset), (e_4, \emptyset), (e_5, \emptyset)\}$ be soft set over $\mathcal{U}$ and $R_s = \{(\emptyset, e_2), (\emptyset, e_3), (\emptyset, e_4), (\emptyset, e_5)\}$ be an any binary relation over $\mathcal{U} \times E$. Let $H_B = \{(e_2, \emptyset), (e_3, \emptyset)\} = \emptyset$. then the null soft matrix is as follows:

$$[a_{ij}] = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, Hence \quad \tilde{\tau}_{R_s}(H_B) = \{\tilde{\mathcal{U}}, \tilde{\emptyset}\}.$$

(ii) Let $\mathcal{U} = \{d_1, d_2, d_3, d_4\}$be the four dresses and $E = \{costly\} = \{e_1\}$ contains only one parameter. Let A = $\{e_1\} = E$ be soft set over $\mathcal{U}$ and $F_A = \{(e_1, \{d_1, d_2, d_3, d_4\})\}$,$R_s = \{(d_1, e_1), (d_2, e_1), (d_3, e_1), (d_4, e_1)\}$,$R_s(e_1) = \{d_1, d_2, d_3, d_4\}$ be an any binary relation over $\mathcal{U} \times E$. Let H_B=$(e_1, \{d_4\})$ then the column soft matrix is as follows:

$$[a_{ij}] = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \end{pmatrix}, Hence \quad \tilde{\tau}_{R_s}(H_B) = \{\tilde{\mathcal{U}}, \tilde{\emptyset}\}.$$

(iii) Let $\mathcal{U} = \{d_1\}$ be the only one dress and $E = \{costly,\ beautiful,\ cheap,\ comfortable\} = \{e_1, e_2, e_3, e_4\}$. Let A = $\{e_2,\ e_3, e_4\} \subseteq E$ be soft set over $\mathcal{U}$ and $F_A = \{(e_2,\ \{d_1\}), (e_3, \{d_1\}),\ (e_4, \{d_1\})\}$ and $H_B = \{(e_2,\ \{d_1\})\}$ $R_s = \{(\mathcal{U}, e_2),\ (\mathcal{U}, e_3),\ (\mathcal{U}, e_4)\}$ and $R_s(e_2) = \{d_1\}$,$R_s(e_3) = \{d_1\}$,$R_s(e_4) = \{d_1\}$ be an binary relation over $\mathcal{U} \times E$. Then the row soft matrix is as follows:

$$[a_{ij}] = \begin{pmatrix} 0 & 1 & 1 & 1 \end{pmatrix}, Hence \quad \tilde{\tau}_{R_s}(H_B) = \{\tilde{\mathcal{U}}, \tilde{\emptyset}, (e_2, \{d_1\})\}.$$

(iv) Let $\mathcal{U} = \{d_1,\ d_2, d_3, d_4, d_5\}$ be the five dresses and $E = \{costly,\ beautiful,\ cheap, comfortable,\ gorgeous\} = \{e_1, e_2, e_3,\ e_4, e_5\}$. Let A=$\{e_2,\ e_3, e_4,\ e_5\} \subseteq E$ and $F_A = \{(e_2,\ \{d_1, d_4\}),\ (e_3, \{d_2, d_3\}),\ (e_4, \{d_1,\ d_3\}), (e_5,\ \{d_4, d_5\})$ be soft set over $\mathcal{U}$ and $R_s = \{(d_1,\ e_2), (d_4, e_2),\ (d_2, e_3), (d_3, e_3),\ (d_1, e_4),\ (d_3, e_4),\ (d_4, e_5),\ (d_5, e_5)\}$ and $R_s(e_2) = \{d_1, d_4\}, R_s\ (e_3) = \{d_2, d_3\},\ R_s(e_4) = \{d_1, d_3\},\ R_s(e_5) = \{d_4, d_5\}$ be an binary relation over $\mathcal{U} \times E$. Let H_B=$\{(e_2,\ \{d_1, d_4\}),\ (e_5, \{d_4\})\}$ then the square soft matrix is as follows:

$$[a_{ij}] = \begin{pmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Hence $\tilde{\tau}_{R_s}(H_B) = \{\tilde{\mathcal{U}},\ \tilde{\emptyset}, \{(e_2,\ \{d_1,\ d_4\})\},\ \{(e_2,\ \{d_1,\ d_4\}),\ (e_5, \{d_4,\ d_5\})\},\ \{(e_5,\ \{d_4,\ d_5\})\}\}$.

(v) Let $\mathcal{U} = \{d_1, d_2, d_3, d_4\}$ be the four dresses and $E = \{costly, beautiful, cheap, comfortable, gorgeous\} = \{e_1,\ e_2, e_3,\ e_4, e_5\}$. Let A=$\{e_2,\ e_3, e_4,\ e_5\} \subseteq E$ and $F_A = \{(e_2,\ \mathcal{U}),\ (e_3, \mathcal{U}),\ (e_4, \mathcal{U}),\ (e_5, \mathcal{U})\}$ be soft set over $\mathcal{U}$ and $R_s = \{(\mathcal{U}, e_2),$

$(\mathcal{U}, e_3)$, $(\mathcal{U}, e_4)$, $(\mathcal{U}, e_5)\}$ be an binary relation over $\mathcal{U} \times E$. Let H_B=$\{(e_2$, $\{d_1, d_2\})$, $(e_3, \{d_1\})$, $(e_4, \mathcal{U})\}$ then the complete soft matrix is as follows:

$$[a_{ij}] = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Hence $\tilde{\tau}_{R_s}(H_B) = \{\tilde{\mathcal{U}}, \tilde{\emptyset}, \{(e_4, \{d_1, d_2, d_3, d_4\})\}, \{(e_2, \{d_1, d_2, d_3, d_4\}), (e_3, \{d_1,$ $d_2, d_3, d_4\})$, $(e_2, \{d_1, d_2, d_3, d_4\})$, $\{(e_2, \{d_1, d_2, d_3, d_4\})$, $(e_3, \{d_1, d_2, d_3, d_4\})\}\}$.

(vi) Let $\mathcal{U} = \{d_1, d_2, d_3, d_4, d_5\}$ be the five dresses and $E = \{costly,\ beautiful,$ $cheap,\ comfortable,\ gorgeous\} = \{e_1\ , e_2, e_3,\ e_4, e_5\}$. Let A=$\{e_1,\ e_2, e_3,$ $e_4, e_5\} \subseteq E$ and $F_A = \{(e_1, \{d_1\}), (e_2, \{d_2\}), (e_3, \{d_3\}), (e_4, \{d_4\}), (e_5, \{d_5\})\}$ be soft set over $\mathcal{U}$ and $R_s = \{(d_1, e_1), (d_2, e_2), (d_3, e_3), (d_4, e_4), (d_5, e_5)\}$ and $R_s(e_1) = \{d_1\}$, $R_s(e_2) = \{d_2\}$, $R_s(e_3) = \{d_3\}, R_s(e_4) = \{d_4\}$, $R_s(e_5) = \{d_5\}$ be an binary relation over $\mathcal{U} \times E$. Let H_B=$\{(e_1, \{d_1\}), (e_2, \{d_2\})\}$, then the diagonal soft matrix is as follows:

$$[a_{ij}] = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, Hence \quad \tilde{\tau}_{R_s}(H_B) = \{\tilde{\mathcal{U}}, \tilde{\emptyset}, \{(e_1, \{d_1\}), (e_2, \{d_2\})\}\}.$$

Definition 4.6: Let $(\mathcal{U}, F_A, R_s)$ be an triple order pair of soft approximation space induced by binary relation. Let $[a_{ij}], [b_{ij}] \in SM_{m\times n}$ Then the soft matrix is called

(i) The union of soft matrix of $[a_{ij}]$ and $[b_{ij}]$,denoted by $[a_{ij}] \cup [b_{ij}]$, If $C_{ij} = max\{a_{ij}, b_{ij}\}$.

(ii) The intersection of soft matrix of $[a_{ij}]$ and $[b_{ij}]$, denoted by $[a_{ij}] \cap [b_{ij}]$, If $C_{ij} = min\{a_{ij}, b_{ij}\}$

(i) The complement of soft matrix of $[a_{ij}]$ denoted by$[a_{ij}]^c$, If $C_{ij} = 1 - a_{ij}$.

Example 4.7: Let $\mathcal{U} = \{u_1,\ u_2, u_3, u_4\}$, $E = \{e_1, e_2,\ e_3, e_4\}$ and $A = \{e_1, e_2\} \subseteq E$. Let $F_A = \{(e_1,\ \{u_2, u_4\}),\ (e_2, \{u_1, u_3\})\}$ and $H_B = \{(e_1,\ \{u_2\}), (e_2,\ \{u_1, u_3\})\}$.

Then $R_s = \{(u_2,\ e_1), (u_4, e_1),\ (u_1, e_2), (u_3, e_2)\}$ and $R_s(e_1) = \{u_2, u_4\}$, $R_s(e_2) = \{u_1, u_3\}$. We have

$$[a_{ij}] = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, [b_{ij}] = \begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

Hence $\tilde{\tau}_{R_s}\ (H_B) = \{\tilde{\mathcal{U}},\ \tilde{\emptyset},\ \{(e_2, \{u_1,\ u_3\})\},\ \{(e_1,\ \{u_2\})\}\}$. If $B = \{e_2, e_3,\ e_4\} \subseteq E$. Let $F_B = \{(e_2,\ \{u_2\}), (e_3,\ \{u_1, u_4\}), (e_4,\ \mathcal{U})\}$ and $H_C = \{(e_2, \{u_2\}),\ (e_4, \{u_1,\ u_2\})\}$. Then $R_s = \{(u_2, e_2)\ , (u_1, e_3),\ (u_4, e_3),\ (\mathcal{U}, e_4)\}$ and $R_s(e_2) = \{u_2\}, R_s(e_3) = \{u_1, u_4\}$, $R_s(e_4) = \mathcal{U}$. Hence $\tilde{\tau}_{R_s}(H_C) = \{\tilde{\mathcal{U}}, \tilde{\emptyset}, \{(e_2,\ \{u_2\})\},\ \{(e_2, \{u_2\}),\ (e_4, \{u_1,\ u_2\})\}, \{(e_4,\ \{u_1, u_2\})\}\}$. Also we have $C_1 = A \cup B$ and $F_A \cup F_B = F_{C_1} = \{(e_1, \{u_2,\ u_4\}), (e_2,\ \{u_1, u_3\}),\ (e_2, \{u_2\}),\ (e_3, \{u_1, u_4\}),\ (e_4, \mathcal{U})\}$ and $H_D = \{(e_1,\ \{u_2, u_4\}),\ (e_2, \{u_2\}),\ (e_3, \{u_1,\ u_4\})\}$. Then $R_s = \{(u_2,\ e_1), (u_4,\ e_1), (u_1,\ e_2), (u_3,\ e_2), (u_2,\ e_2),\ (u_1, e_3),\ (u_4, e_3)\ , (\mathcal{U}, e_4)\}$ and $R_s(e_1) = \{u_2,\ u_4\}, R_s\ (e_2) = \{u_1,\ u_2, u_3\}$, $R_s(e_3) = \{u_1, u_4\}$, $R_s(e_4) = \{u_1, u_2,\ u_3, u_4\}$. Similarly, we have $C_2 = A \cap B$ and $F_A \cap F_B = F_{C_2} = \emptyset$ and $H_{D'} = \emptyset$. Also we have complement of soft matrix $C_3 = 1 - A, F_{C_3} = (F_A)^c = \{(e_1,\ \{u_1, u_3\}),\ (e_2, \{u_2,\ u_4\}), (e_3,\ \mathcal{U}),\ (e_4, \mathcal{U})\}$, and $H_{D^c} = \{(e_1,\ \{u_1,\ u_3\}), (e_2,\ \{u_2\}), (e_3,\ \{u_1\}),\ (e_4, \{u_2\})\}$

$$[a_{ij}] \cup [b_{ij}] = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{pmatrix}, [a_{ij}] \cap [b_{ij}] = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, [a_{ij}]^c = \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{pmatrix}$$

Hence $\tilde{\tau}_{R_s}(H_D) = \{\tilde{\mathcal{U}},\ \tilde{\emptyset}, \{(e_1,\ \{u_2, u_4\}),\ (e_3, \{u_1, u_4\})\},\ \{(e_1, \{u_2, u_4\}),\ (e_2, \{u_1,\ u_2, u_3\}),\ (e_3, \{u_1,\ u_4\})\},\ \{(e_2, \{u_1,\ u_2, u_3\})\}$ also $\tilde{\tau}_{R_s}(H_{D'}) = \{\tilde{\mathcal{U}},\ \tilde{\emptyset}\}$ and $\tilde{\tau}_{R_s}(H_{D^c}) = \{\tilde{\mathcal{U}},\ \tilde{\emptyset}, \{(e_1, \{u_1,\ u_3\})\}, \{(e_2,\ \{u_2, u_4\}),\ (e_3, \mathcal{U})\},\ (e_4, \{u_1,\ u_2, u_3,\ u_4\})\}$

Proposition 4.8: Let $(\mathcal{U}, F_A, R_s)$ be an soft approximation space induced by any binary relation and $[a_{ij}], [b_{ij}] \in SM_{m\times n}$. Then

(i) $[[a_{ij}]^c]^c = [a_{ij}]$ where $[a_{ij}]^c = 1 - [a_{ij}]$.

(ii) $[0]^c = [1]$.

(iii) $[a_{ij}] \cup [a_{ij}] = [a_{ij}]$.

(iv) $[a_{ij}] \cup [0] = [a_{ij}]$ and $[a_{ij}] \cup [1] = [1]$

(v) $[a_{ij}] \cup [b_{ij}] = [b_{ij}] \cup [a_{ij}]$.

(vi) $[a_{ij}] \cap [a_{ij}] = [a_{ij}]$

(vii) $[a_{ij}] \cap [0] = [0]$ and $[a_{ij}] \cap [1] = [a_{ij}]$

Proposition 4.9: Let $(\mathcal{U}, F_A, R_s)$ be an soft approximation space induced by any binary relation and $[a_{ij}], [b_{ij}] \in SM_{m\times n}$. Then

(i) $([a_{ij}] \cup [b_{ij}])^c = [a_{ij}]^c \cap [b_{ij}]^c$.

(i) $([a_{ij}] \cap [b_{ij}])^c = [a_{ij}]^c \cup [b_{ij}]^c$

Proof:

(i) Let $[a_{ij}], [b_{ij}] \in SM_{m\times n}$. We know that $[a_{ij}] \cup [b_{ij}]$, which is equal to $max\{a_{ij}, b_{ij}\}$, by definition. Now we claim that $([a_{ij}] \cup [b_{ij}])^c$. It follows by definition 4.5, $[max\{a_{ij}, b_{ij}\}]$, then $[1 - max\{a_{ij}, b_{ij}\}]$, which implies that $[min\{1 - a_{ij}, 1 - b_{ij}\}]$. We can obtain that $[min\{1 - a_{ij}\}], min\{1 - b_{ij}\}$. Hence $[a_{ij}]^c \cap [b_{ij}]^c = [a_{ij}]^c \cup [b_{ij}]^c$.

(ii) The proof is similarly proved by proof (i).

5. Application

In this section, we discuss the comparative analysis between nano topological space and nano soft topological space induced by soft matrix, then the concepts are applied in real life application of "life expectancy". It is well known that the computation of soft matrix lower and soft matrix upper approximation and soft matrix boundary region of an soft approximation space induced by relation $(\mathcal{U}, F_A, R_s)$.The approximation space of granulation structure induced by a soft subset is referred to as a soft approximation space. We will show the above discussion by the following example.

Example 5.1: Let us consider the universe $\mathcal{U} = \{u_1, u_2, u_3, u_4, u_5, u_6\}$ consists of six persons and $E = \{e_1, e_2, e_3, e_4\}$ is a set of decision parameters, which describes the "Life expectancy". The e_1 stands for "Under stress", e_2 stands for "young", e_3 stands for "Drug addict" and e_4 stands for "Healthy". Let $F_A = \{(e_1, \{u_5\}), (e_2, \{u_1, u_2\}), (e_3, \{u_4\}), (e_4, \{u_1, u_2, u_3, u_6\})\} = H_B$ and $R_s = \{(u_5, e_1), (u_1, e_2), (u_2, e_2), (u_4, e_3), (u_1, e_4), (u_2, e_4), (u_3, e_4), (u_6, e_4)\}$. Then, $R_s(e_1) = \{u_5\}$,

$R_s(e_2) = \{u_1, u_2\}$, $R_S(e_3) = \{u_4\}, R_s\ (e_4) = \{u_1, u_2,\ u_3, u_6\}$, be any binary relation induced by soft set. Then the soft matrix defined as follows:

$$[a_{ij}] = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Let us calculate the lower approximation of soft matrix, upper approximation of soft matrix and boundary region of soft matrix as follows: Here

(i) Let $H_B = \{(e_4,\ \{u_1, u_2,\ u_3, u_6\})\}$, then it follows that $L_{R_s}(H_B) = \{(e_4, \{u_1,\ u_2, u_3, u_6\})\}$, $U_{R_s}(H_B) = \{(e_4, \{u_1,\ u_2, u_3,\ u_6\})\}$, $B_{R_s}\ (H_B) = \tilde{\emptyset}$ ie., $\tilde{\tau}_{R_s}\ (H_B) = \{\tilde{\mathcal{U}}, \tilde{\emptyset}, \{(e_4, \{u_1,\ u_2, u_3,\ u_6\})\}$

(ii) let $H_B = \{(e_1,\ \{u_5\})\}$, then it follows that $L_{R_s}(H_B) = (e_1, \{u_5\})$, $U_{R_s}(H_B) = \{(e_1, \{u_5\})\}$, $B_{R_s}(X) = \tilde{\emptyset}$ ie., $\tau_{R_s}(H_B) = \{\tilde{\mathcal{U}}, \tilde{\emptyset}, \{(e_1,\ \{u_5\})\}\}$

(iii) let $H_B = \{(e_1,\ \{u_5\}), (e_3,\ \{u_4\})\} \subseteq F_A$, it follows that $L_{R_s}(H_B) = H_B = \{(e_1, \{u_5\}),\ (e_3, \{u_4\})\}$, $U_{R_s}(H_B) = H_B = \{(e_1, \{u_5\}),\ (e_3, \{u_4\})\}$ and $B_{R_s}\ (H_B) = \tilde{\emptyset}$. Hence $\tilde{\tau}_R(H_B) = \{\tilde{\mathcal{U}}, \tilde{\emptyset}, \{(e_1,\ \{u_5\}),\ (e_3, \{u_4\ \})\}\}$.

(iv) let $H_B = \{(e_4,\ \{u_2,\ u_3\})\}$, then it follows that $L_{R_s}(H_B) = \tilde{\emptyset}$, $U_{R_s}(H_B) = \{(e_4,\ \{u_2, u_3\})\}$, $B_{R_s}\ (H_B) = \{(e_4,\ \{u_2,\ u_3\})\}$ ie., $\tilde{\tau}_{R_s}\ (H_B) = \{\tilde{\mathcal{U}}, \tilde{\emptyset}, \{(e_4,\ \{u_2,\ u_3\})\}\}$

On the other hand ,"Life expectancy" topic can also be described using nano topological space as follows: The evaluation will be done in terms of attributes as "Sex", "age category", "Living area", "Habits". Then the characterized by the value sets "{men,woman}", "{baby, young, mature age,old}","{village,city}", "{smoke,drinking, smoke and drinking, no smoke and no drinking}". We denote "smoke and drinking" by SD and "no smoke and no drinking" by NSND. The information will be given by following table, where the rows are labled by attributes and the table entries are the attribute values for each person. Here we obtain the following equivalence relation, induced by the above mentioned attributes as follows: $\mathcal{U}/R = \{\{u_1, u_2\}, \{u_3, u_5\}, \{u_4\}, \{u_6\}\}$. Let X be a subset of $\mathcal{U}$,then we wish to represent using the above equivalence relation. Hence we analyze the lower,upper approximation space and boundary region of X in some particular cases as follows:

(i) Let X=$\{u_1, u_2, u_3, u_6\}$, it follows that $L_R(X) = \{u_1, u_2, u_6\}, U_R(X) = \{u_1, u_2, u_3, u_5, u_6\}$ and $B_R(X) = \{u_3, u_5\}$. Hence $\tau_R(X) = \{\mathcal{U}, \emptyset, \{u_1, u_2, u_6\}, \{u_1, u_2, u_3, u_5, u_6\}, \{u_3, u_5\}\}$.

(ii) Let X=$\{u_5\}$. It follows that $L_R(X) = \emptyset$, $U_R(X) = \{u_3, u_5\}$ and $B_R(X) = \{u_3, u_5\}$. Hence $\tau_R(X) = \{\mathcal{U}, \emptyset, \{u_3, u_5\}\}$.

(iii) Let X=$\{u_4, u_5\}$. It follows that $L_R(X) = \{u_4\}, U_R(X) = \{u_3, u_4, u_5\}$ and $B_R(X) = \{u_3, u_5\}$. Hence $\tau_R(X) = \{\mathcal{U}, \emptyset, \{u_4\}, \{u_3, u_4, u_5\}, \{u_3, u_5\}\}$.

(iv) Let X=$\{u_2, u_3\}$. It follows that $L_R(X) = \emptyset$, $U_R(X) = \{u_1, u_2, u_3, u_5\}$ and $B_R(X) = \{u_1, u_2, u_3, u_5\}$. Hence $\tau_R(X) = \{\mathcal{U}, \emptyset, \{u_1, u_2, u_3, u_5\}\}$.

Comparison between the approximation space$(\mathcal{U}, R)$ and the soft approximation space $(\mathcal{U}, F_A, R_s)$

X& H_B	Approximation Space [5]			Soft Approximation Space		
	$L_R(X)$	$U_R(X)$	$B_R(X)$	$L_{R_s}(H_B)$	$U_{R_s}(H_B)$	$B_{R_s}(H_B)$
$\{u_1, u_2, u_3, u_6\}$ & $\{(e_4, \{u_1, u_2, u_3, u_6\})\}$	$\{u_1, u_2, u_6\}$	$\{u_1, u_2, u_3, u_5, u_6\}$	$\{u_3, u_5\}$	H_B	H_B	$\tilde{\emptyset}$
$\{u_5\}$ &$\{(e_1, \{u_5\})\}$	$\emptyset$	$\{u_3, u_5\}$	$\{u_3, u_5\}$	H_B	H_B	$\tilde{\emptyset}$
$\{u_4, u_5\}$ & $\{(e_1, \{u_5\}), (e_3, \{u_4\})\}$	$\{u_4\}$	$\{u_3, u_4, u_5\}$	X	H_B	H_B	$\tilde{\emptyset}$
$\{u_2, u_3\}$ &$\{(e_4, \{u_2, u_3\})\}$	$\emptyset$	$\{u_1, u_2, u_3, u_5\}$	U_R	$\emptyset$	H_B	H_B

Observation: Here the above results show that soft approximation space stimulated by relation $(\mathcal{U}, F_A, R_S)$ method reduce the soft boundary region and increasing the soft lower and soft upper approximations with comparison of approximation space [5]. The nano soft topology induced by soft matrix is a worth considering alternative to the nano topology in approximation space $(\mathcal{U}, R)$. Soft set based on nano topological space could provide a better approximation space than nano topological space.

6. Conclusion

This paper systematically proposed the new concept of nano topological space induced by soft set, the **"Nano Soft Topological Space(NSTS)"**. The properties of soft lower and soft upper approximations are studied with examples. Further, we examined the soft matrix ie., null soft matrix,square soft matrix etc., stimulated based on nano soft topological space. An example containing a comparative analysis between approximation space and soft approximation space was discussed.

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