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ELASTODYNAMIC RESPONSE OF AN ANISOTROPIC MEDIUM IN FREQUENCY DOMAIN

Anita Goel, Surendra Kumar, N. R. Garg and Ritu Goel

Department of Mathematics
Maharshi Dayanand University, Rohtak-124001, India

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Abstract: Two-dimensional elastodynamic displacements and stresses for a monoclinic solid have been obtained in frequency domain in relatively simple form. The eigenvalue method, following Fourier transform, has been used to obtain the response in the transformed domain. The analytic eigenvalue method for a monoclinic solid, presented in this paper, is straightforward and is convenient for numerical computation. The use of matrix notation avoids unwieldy mathematical expressions. A particular case of a normal line-load acting in the interior of an infinite orthotropic solid has been considered in detail. The corresponding deformation in frequency domain is obtained numerically. The variations of dimensionless displacements and stresses with the horizontal distance have been shown graphically.

Keywords and Phrases: Two-dimensional deformation, anisotropy, frequency domain, eigenvalue method
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1. Introduction

The displacement Green's functions for a homogeneous isotropic linearly elastic solid are classical results in elastodynamic theory. Stokes [26] read a paper on the dynamical theory of diffraction. He obtained an exact solution for the displacement field due to a tangential force in an infinite elastic solid. Thus, without knowing it, he had conceived the first mathematical model of an earthquake. Later on, Lamb [12] presented a paper "On the propagation of tremors over the surface of an elastic solid". Soon afterwards, Love [14] studied a paper on "The propagation of wave-motion in an isotropic elastic solid medium". The Stokes-Love formulation assumed a localized harmonic force in an elastic isotropic medium. By differentiation and addition, Love [15] obtained the dynamical response to dipoles, couples, double couples, explosions, etc. The three-dimensional elastodynamic result for isotropic solids has been also obtained, amongst others, by Maruyama [16], Achenbach [1], Niwa *et al.* [19] and Karabolis and Beskos

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[11]. The dislocation problems of shear and tensile faults within the earth, modelled as isotropic solid, have received great attention in the geophysical literature from many investigators, namely, Maruyama [17], Okada [20,21], Bonafede and Rivalta [3,4].

Dziewonski and Anderson [8] established that the upper part of the earth is anisotropic. The corresponding solutions for anisotropic solids have also received a great deal of attention in the past. Unfortunately, explicit expressions for displacement Green's functions of anisotropic materials are very difficult to obtain, both in the frequency domain and the time domain. Buchwald [5] discussed the problem of elastic wave propagation in a transversely isotropic elastic infinite media. in frequency domain due to a point source. For a transversely isotropic medium, some other relevant contributions are those of Payton [23], Yeatts [29] and Pan [22].

The work by Lighthill [13] for asymptotic solution and by Duff [7] on integral expressions for the Cauchy problem are the early treatments on general anisotropy by the use of Fourier transform. Tverdokhlebov and Rose [27] and Wang and Achenbach [28] used Radon transform to study elastodynamic problems in an anisotropic medium.

The orthotropy symmetry is exhibited by olivine and orthopyroxenes, the principal rock-forming minerals of deep crust and upper mantle. Garg *et al.* [10] obtained the representation of seismic sources causing static antiplane strain deformation of an orthotropic medium. The corresponding static plane strain deformation of an orthotropic elastic medium has been discussed by Garg *et al.* [9].

In the present paper, a novel analytical eigenvalue method is presented for the elastodynamic deformation of a monoclinic solid in frequency domain. The Fourier transformation on the equations of motion for plane strain deformation of a monoclinic solid reduces them into a single linear homogeneous vector-matrix differential equation of second order, on which an eigenvalue method is applied to obtain a solution in the transformed domain. The procedure developed in this paper is relatively simple, straightforward, avoids the cumbersome nature of the problem and is convenient for numerical computation. The form of the single governing equation for a monoclinic solid, derived in this paper, is a new contribution. As a particular case, a normal line-load acting in the interior of an infinite orthotropic solid has been considered in detail. The solution so obtained for an orthotropic solid in the transform domain is analytic. The displacements and stresses in the integral form have been inverted numerically. The variations of dimensionless displacements and stresses with the horizontal distance have been shown graphically.

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The analytical eigenvalue method developed in this paper may also be applied to two-dimensional sources such as strip-load. The deformation at any point of the medium is useful to analyse the deformation field around mining tremors and drilling into the crust of the earth. It may also find applications in various engineering problems, crystal physics and solid-earth geophysics regarding the deformation of an anisotropic solid. The present paper may be considered as an improvement of our pervious paper Garg *et al.* [9] in two ways, firstly, we have moved from static problem to dynamic problem in the frequency domain and secondly, the medium is more anisotropic, i.e., monoclinic, instead of orthotropic. The results for an isotropic medium are derivable as a special case of Garg *et al.* [9] and are in agreement with the earlier results of Love [15] and Maruyama [17].

2. Basic Equations

The equations of motion in the Cartesian co-ordinate system (x_1, x_2, x_3) in the absence of body force are

$$\frac{\partial \tau_{11}}{\partial x_1} + \frac{\partial \tau_{12}}{\partial x_2} + \frac{\partial \tau_{13}}{\partial x_3} = \rho \frac{\partial^2 u_1}{\partial t^2}, \tag{2.1}$$

$$\frac{\partial \tau_{21}}{\partial x_1} + \frac{\partial \tau_{22}}{\partial x_2} + \frac{\partial \tau_{23}}{\partial x_3} = \rho \frac{\partial^2 u_2}{\partial t^2}, \tag{2.2}$$

$$\frac{\partial \tau_{31}}{\partial x_1} + \frac{\partial \tau_{32}}{\partial x_2} + \frac{\partial \tau_{33}}{\partial x_3} = \rho \frac{\partial^2 u_3}{\partial t^2}, \tag{2.3}$$

where τ_{ij} $(i, j = 1, 2, 3)$ are the components of the stress tensor, (u_1, u_2, u_3) are the components of the displacement vector and ρ is the density of the elastic medium.

The stress-strain relations for a monoclinic elastic medium, with x_1x_2-plane as plane of elastic symmetry, are Atanackovic and Guran [2].

$$\tau_{11} = d_{11}e_{11} + d_{12}e_{22} + d_{13}e_{33} + 2d_{16}e_{12}, \tag{2.4}$$

$$\tau_{22} = d_{12}e_{11} + d_{22}e_{22} + d_{23}e_{33} + 2d_{26}e_{12}, \tag{2.5}$$

$$\tau_{33} = d_{13}e_{11} + d_{23}e_{22} + d_{33}e_{33} + 2d_{36}e_{12}, \tag{2.6}$$

$$\tau_{23} = 2d_{44}e_{23} + 2d_{45}e_{13}, \tag{2.7}$$

$$\tau_{13} = 2d_{45}e_{23} + 2d_{55}e_{13}, \tag{2.8}$$

$$\tau_{12} = d_{16}e_{11} + d_{26}e_{22} + d_{36}e_{33} + 2d_{66}e_{12}, \tag{2.9}$$

where e_{ij} $(i, j = 1, 2, 3)$ are the components of strain tensor and are related with displacement components (u_1, u_2, u_3) through the relations

$$e_{ij} = \frac{1}{2}\left[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right], \quad 1 \leq i, j \leq 3. \tag{2.10}$$

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The two-suffix symmetric quantities d_{ij} $(i,j = 1,2,\cdots,6)$ are the elastic moduli for the monoclinic elastic medium.

For an orthotropic solid Mase and Mase [18] with coordinate planes coinciding with the planes of symmetry, we have

$$d_{16} = d_{26} = d_{36} = d_{45} = 0, \tag{2.11}$$

and the number of independent elastic coefficients reduces to nine. A transversely isotropic medium is a particular case of orthotropy in which

$$d_{22} = d_{11}, \quad d_{23} = d_{13}, \quad d_{55} = d_{44}, \quad d_{66} = \frac{1}{2}(d_{11} - d_{12}) \tag{2.12a}$$

and there are five independent elastic coefficients. Further, isotropy is also a particular case of an orthotropy in which

$$d_{11} = d_{22} = d_{33} = \lambda + 2\mu, \quad d_{12} = d_{13} = d_{23} = \lambda, \quad d_{44} = d_{55} = d_{66} = \mu, \tag{2.12b}$$

where λ and μ are Lame's elastic moduli.

For convenience, we shall write $(x_1, x_2, x_3) = (x, y, z)$ and

$$(u_1, u_2, u_3) = (u, v, w).$$

The equations of motion for a monoclinic elastic medium in terms of displacement components can be obtained from equations (2.1)-(2.10). We find

$$\left[d_{11}\frac{\partial^2 u}{\partial x^2} + d_{66}\frac{\partial^2 u}{\partial y^2} + d_{55}\frac{\partial^2 u}{\partial z^2} + 2d_{16}\frac{\partial^2 u}{\partial x \partial y}\right] + \left[d_{16}\frac{\partial^2 v}{\partial x^2} + d_{26}\frac{\partial^2 v}{\partial y^2} + d_{45}\frac{\partial^2 v}{\partial z^2}\right.$$

$$\left. + (d_{12} + d_{66})\frac{\partial^2 v}{\partial x \partial y}\right] + \left[(d_{13} + d_{55})\frac{\partial^2 w}{\partial x \partial z} + (d_{36} + d_{45})\frac{\partial^2 w}{\partial y \partial z}\right] = \rho\frac{\partial^2 u}{\partial t^2}, \tag{2.13}$$

$$\left[d_{16}\frac{\partial^2 u}{\partial x^2} + d_{26}\frac{\partial^2 u}{\partial y^2} + d_{45}\frac{\partial^2 u}{\partial z^2} + (d_{66} + d_{12})\frac{\partial^2 u}{\partial x \partial y}\right] + \left[d_{66}\frac{\partial^2 v}{\partial x^2} + d_{22}\frac{\partial^2 v}{\partial y^2} + d_{44}\frac{\partial^2 v}{\partial z^2}\right.$$

$$\left. + 2d_{26}\frac{\partial^2 v}{\partial x \partial y}\right] + \left[(d_{36} + d_{45})\frac{\partial^2 w}{\partial x \partial z} + (d_{23} + d_{44})\frac{\partial^2 w}{\partial y \partial z}\right] = \rho\frac{\partial^2 v}{\partial t^2}, \tag{2.14}$$

$$\left[(d_{55} + d_{13})\frac{\partial^2 u}{\partial x \partial z} + (d_{45} + d_{36})\frac{\partial^2 u}{\partial y \partial z}\right] + \left[(d_{45} + d_{36})\frac{\partial^2 v}{\partial x \partial z} + (d_{44} + d_{23})\frac{\partial^2 v}{\partial y \partial z}\right]$$

$$+ \left[d_{55}\frac{\partial^2 w}{\partial x^2} + d_{44}\frac{\partial^2 w}{\partial y^2} + d_{33}\frac{\partial^2 w}{\partial z^2} + 2d_{45}\frac{\partial^2 w}{\partial x \partial y}\right] = \rho\frac{\partial^2 w}{\partial t^2}. \tag{2.15}$$

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We now consider the plane-strain deformation, parallel to the xy-plane, in which the displacement components are independent of z and are of the type

$$u = u(x,y,t), \quad v = v(x,y,t), \quad w \equiv 0. \tag{2.16}$$

The non-zero stresses for the plane-strain problem for a monoclinic medium are to be obtained from equations (2.4)-(2.10) and (2.16). We find

$$\tau_{11} = d_{11}\frac{\partial u}{\partial x} + d_{12}\frac{\partial v}{\partial y} + d_{16}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \tag{2.17}$$

$$\tau_{22} = d_{12}\frac{\partial u}{\partial x} + d_{22}\frac{\partial v}{\partial y} + d_{26}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \tag{2.18}$$

$$\tau_{33} = d_{13}\frac{\partial u}{\partial x} + d_{23}\frac{\partial v}{\partial y} + d_{36}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \tag{2.19}$$

$$\tau_{12} = d_{16}\frac{\partial u}{\partial x} + d_{26}\frac{\partial v}{\partial y} + d_{66}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right). \tag{2.20}$$

The equations of motion, given in equations (2.13)-(2.14), for plane-strain deformation (parallel to xy-plane) of a monoclinic elastic medium are

$$\left[d_{11}\frac{\partial^2 u}{\partial x^2} + d_{66}\frac{\partial^2 u}{\partial y^2} + 2d_{16}\frac{\partial^2 u}{\partial x \partial y}\right] + \left[d_{16}\frac{\partial^2 v}{\partial x^2} + d_{26}\frac{\partial^2 v}{\partial y^2} + (d_{12}+d_{66})\frac{\partial^2 v}{\partial x \partial y}\right] = \frac{\rho \partial^2 u}{\partial t^2}, \tag{2.21a}$$

$$\left[d_{16}\frac{\partial^2 u}{\partial x^2} + d_{26}\frac{\partial^2 u}{\partial y^2} + (d_{66}+d_{12})\frac{\partial^2 u}{\partial x \partial y}\right] + \left[d_{66}\frac{\partial^2 v}{\partial x^2} + d_{22}\frac{\partial^2 v}{\partial y^2} + 2d_{26}\frac{\partial^2 v}{\partial x \partial y}\right] = \frac{\rho \partial^2 v}{\partial t^2}, \tag{2.21b}$$

and equation (2.15) reduces to an identity.

We assume that the disturbances are harmonic time-dependent ($e^{i\omega t}$) and we take the solution of system of simultaneous equations (2.21a), (2.21b) in the form

$$\begin{aligned} u(x,y,t) &= U(x,y)e^{i\omega t}, \\ v(x,y,t) &= V(x,y)e^{i\omega t}, \end{aligned} \tag{2.22}$$

where ω is the angular frequency of the vibration. For convenience, we write

$$\tau_{11}(x,y,t) = T_{11}(x,y)e^{i\omega t}, \quad \tau_{12}(x,y,t) = T_{12}(x,y)e^{i\omega t}, \quad \tau_{22} = T_{22}(x,y)e^{i\omega t}. \tag{2.23}$$

The assumption of harmonic time-dependence simplifies the calculations to a greater extent and one gets a solution in a frequency domain.

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Substituting of the values of u and v from equation (2.22) in equations (2.21a,b), we get

$$\left[d_{11}\frac{\partial^2 U}{\partial x^2}+d_{66}\frac{\partial^2 U}{\partial y^2}+2d_{16}\frac{\partial^2 U}{\partial x\partial y}\right]+\left[d_{16}\frac{\partial^2 V}{\partial x^2}+d_{26}\frac{\partial^2 V}{\partial y^2}\right.$$
$$\left.+(d_{12}+d_{66})\frac{\partial^2 V}{\partial x\partial y}\right]+\rho\omega^2 U \;=\; 0, \tag{2.24}$$
$$\left[d_{16}\frac{\partial^2 U}{\partial x^2}+d_{26}\frac{\partial^2 U}{\partial y^2}+(d_{66}+d_{12})\frac{\partial^2 U}{\partial x\partial y}\right]+\left[d_{66}\frac{\partial^2 V}{\partial x^2}+d_{22}\frac{\partial^2 V}{\partial y^2}\right.$$
$$\left.+2d_{26}\frac{\partial^2 V}{\partial x\partial y}\right]+\rho\omega^2 V \;=\; 0. \tag{2.25}$$

3. Method of Solution

we define the following non-dimensional quantities

$$\alpha \;=\; x/h, \quad \beta \;=\; y/h, \quad \hat{U} \;=\; U/h, \quad \hat{V} \;=\; V/h,$$
$$\sigma_{11} \;=\; T_{11}/d_{66}, \quad \sigma_{12} \;=\; T_{12}/d_{66}, \quad \sigma_{22} \;=\; T_{22}/d_{66},$$
$$a \;=\; d_{11}/d_{66}, \quad b \;=\; d_{22}/d_{66}, \quad c \;=\; d_{12}/d_{66},$$
$$d \;=\; d_{26}/d_{66}, \quad e \;=\; d_{16}/d_{66}, \quad \hat{\omega} \;=\; \omega/\omega_0, \tag{3.1}$$

where

$$\omega_0 \;=\; \sqrt{\frac{d_{66}}{\rho h^2}} \tag{3.2}$$

is the "known frequency" and 'h' is a known fixed quantity which has the dimension of length.

Thus the equations of motion, (2.24) and (2.25), in the dimensionless form are

$$\left[a\frac{\partial^2 \hat{U}}{\partial\alpha^2}+\frac{\partial^2 \hat{U}}{\partial\beta^2}+2e\frac{\partial^2 \hat{U}}{\partial\alpha\partial\beta}\right]+\left[e\frac{\partial^2 \hat{V}}{\partial\alpha^2}+d\frac{\partial^2 \hat{V}}{\partial\beta^2}+(c+1)\frac{\partial^2 \hat{V}}{\partial\alpha\partial\beta}\right]+\hat{\omega}^2\hat{U}=0, \tag{3.3}$$

$$\left[e\frac{\partial^2 \hat{U}}{\partial\alpha^2}+d\frac{\partial^2 \hat{U}}{\partial\beta^2}+(1+c)\frac{\partial^2 \hat{U}}{\partial\alpha\partial\beta}\right]+\left[\frac{\partial^2 \hat{V}}{\partial\alpha^2}+b\frac{\partial^2 \hat{V}}{\partial\beta^2}+2d\frac{\partial^2 \hat{V}}{\partial\alpha\partial\beta}\right]+\hat{\omega}^2\hat{V}=0. \tag{3.4}$$

In equations (3.3) and (3.4), α is the dimensionless vertical distance, β is the dimensionless horizontal distance, $\hat{\omega}$ is the dimensionless angular frequency, $\hat{U}$ is the dimensionless vertical displacement component and $\hat{V}$ is the dimensionless horizontal displacement component and $\sigma_{11},\ \sigma_{12},\ \sigma_{22}$ are the dimensionless

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stress components and $a,\ b,\ c,\ d,\ e$ are dimensionless elastic coefficients. For an isotropic solid, we have

$$a \ = \ 2+\alpha_0, \quad b \ = \ 2+\alpha_0, \quad c \ = \ \alpha_0, \quad d \ = \ 0, \quad e \ = \ 0, \tag{3.5}$$

where

$$\alpha_0 \ = \ \lambda/\mu. \tag{3.6}$$

In particular, for a Poission isotropic solid, in which $\lambda = \mu$, we have

$$\alpha_0 \ = \ 1. \tag{3.7}$$

We define Fourier transform $\bar{f}(\alpha, k, \hat{\omega})$ of a function $\bar{f}(\alpha, \beta, \hat{\omega})$ by the relation Debnath [6]

$$\bar{f}(\alpha, k, \hat{\omega}) \ = \ F[\hat{f}(\alpha, \beta, \hat{\omega})] \ = \ \int_{-\infty}^{\infty} \hat{f}(\alpha, \beta, \hat{\omega}) e^{ik\beta}\, d\beta, \tag{3.8}$$

so that

$$\hat{f}(\alpha, \beta, \hat{\omega}) \ = \ \frac{1}{2\pi} \int_{-\infty}^{\infty} \bar{f}(\alpha, k, \hat{\omega}) e^{-ik\beta}\, dk, \tag{3.9}$$

where k is the Fourier transform parameter and in elastodynamic problems it represents the wave number. We know that

$$F\left[\frac{\partial}{\partial\beta}\, \hat{f}(\alpha, \beta, \hat{\omega})\right] \ = \ (-ik)\bar{f}(\alpha, k, \hat{\omega}) \tag{3.10}$$

and

$$F\left[\frac{\partial^2}{\partial\beta^2}\, \hat{f}(\alpha, \beta, \hat{\omega})\right] \ = \ -\, k^2 \bar{f}(\alpha, k, \hat{\omega}). \tag{3.11}$$

Applying the Fourier transformation on equation of motion, (3.3) and (3.4) for a monoclinic elastic medium, we find

$$\left[a\frac{d^2\bar{U}}{d\alpha^2} - 2eik\frac{d\bar{U}}{d\alpha} + (-k^2)\bar{U}\right] + \left[e\frac{d^2\bar{V}}{d\alpha^2} + (1+c)(-ik)\frac{d\bar{V}}{\partial\alpha} + (-dk^2)\bar{V}\right] + \hat{\omega}^2\bar{U} = 0, \tag{3.12}$$

$$\left[e\frac{d^2\bar{U}}{d\alpha^2} + (1+c)(-ik)\frac{d\bar{U}}{d\alpha} + (-k^2 d)\bar{U}\right] + \left[\frac{d^2\bar{V}}{d\alpha^2} - 2dik\frac{d\bar{V}}{d\alpha} + (-bk^2)\bar{V}\right] + \hat{\omega}^2\bar{V} = 0. \tag{3.13}$$

The above two equations (3.12) and (3.13) can be written in the following single vector-matrix homogeneous differential equation

$$A\,\frac{d^2N}{d\alpha^2} + B\,\frac{dN}{d\alpha} + CN \ = \ 0, \tag{3.14}$$

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where

$$A = \begin{bmatrix} a & e \\ e & 1 \end{bmatrix}, \quad B = \begin{bmatrix} -2ike & -ik(1+c) \\ -ik(1+c) & -2ikd \end{bmatrix},$$
$$C = \begin{bmatrix} \hat{\omega}^2 - k^2 & -k^2 d \\ -k^2 d & \hat{\omega}^2 - bk^2 \end{bmatrix}, \quad N = \begin{bmatrix} \bar{U} \\ \bar{V} \end{bmatrix}. \tag{3.15}$$

We note that matrices $A,\ B,\ C$ are all symmetric. Matrix A depends upon dimensionless elastic moduli only while matrices B and C depend upon dimensionless angular frequency $\hat{\omega}$ and dimensionless wave number k. The governing equation (3.14) for a monoclinic solid is a new contribution and its form is relatively simple, and now eigenvalue method shall be applied to it. For this, we seek a solution of matrix equation (3.14) of the form

$$N(\alpha, k) = E(k)e^{s\alpha}, \tag{3.16}$$

where s is a parameter and $E(k)$ is a matrix of the type 2×1.

Substitution of the value of N from equation (3.16) into differential equation (3.14) gives the following characteristic equation

$$a_0 s^4 + 4a_1 s^3 + 6a_2 s^2 + 4a_3 s + a_4 = 0, \tag{3.17}$$

where

$$a_0 = a - e^2, \quad a_1 = \frac{-i(ad - ce)k}{2}, \quad a_2 = \frac{[(a+1)\hat{\omega}^2 - (ab - c^2 - 2c + 2ed)k^2]}{6},$$
$$a_3 = \frac{-[2i(e+d)\hat{\omega}^2 k + 2i(cd - be)k^3]}{4}, \quad a_4 = [\hat{\omega}^4 k - (1+b)\hat{\omega}^2 k^2 + (b - d^2)k^4]. \tag{3.18}$$

To solve biquadratic equation (3.17), we make use of Descartes' method, in which, we remove the second term by using the transformation

$$z^* = a_0 s + a_1. \tag{3.19}$$

Then the equation (3.17) is transformed into

$$Z^{*^4} + 6Hz^{*^2} + 4Gz^* + (a_0^2 I - 3H^2) = 0, \tag{3.20}$$

where

$$H = a_0 a_2 - a_1^2, \quad G = a_0^2 a_3 - 3a_0 a_1 a_2 + 2a_1^3, \quad I = a_0 a_4 - 4a_1 a_3 + 3a_2^2. \tag{3.21}$$

Following the Descartes' method, the left hand side of equation (3.20) can be resolved into a product of two quadratic factors as

$$z^{*^4} + 6Hz^{*^2} + 4Gz^* + (a_0^2 I - 3H^2) = (z^{*^2} + pz^* + q)(z^{*^2} - pz^* + q'), \tag{3.22}$$

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where

$$q + q' \;=\; p^2 + 6H, \tag{3.23}$$

$$p(q' - q) \;=\; 4G, \tag{3.24}$$

$$qq' \;=\; a_0^2 I - 3H^2. \tag{3.25}$$

When $p = 0$, then q and q' are determined from equations (3.23) and (3.24) directly. Otherwise, eliminating q and q' from equations (3.23) to (3.24), we get

$$\xi^3 + 3b_1\xi^2 + 3b_2\xi + b_3 \;=\; 0, \tag{3.26}$$

where

$$\xi \;=\; p^2, \tag{3.27}$$

$$b_1 \;=\; 4H, \quad b_2 \;=\; \frac{4(12H^2 - a_0^2 I)}{3}, \quad b_3 \;=\; -16G^2. \tag{3.28}$$

The equation (3.26) is cubic in ξ and hence can be solved by the Cardan's method. Using the Cardan's method, the equation (3.26) is reduced to

$$\delta^3 + 3H_1\delta + G_1 \;=\; 0, \tag{3.29}$$

where

$$\delta \;=\; \xi + b_1, \quad H_1 \;=\; b_2 - b_1^2, \quad G_1 \;=\; b_3 - 3b_1b_2 + 2b_1^3. \tag{3.30}$$

The roots of the cubic equation (3.29) are

$$\delta \;=\; u_0 + v_0, \tag{3.31}$$

where u_0 and v_0 are given by

$$u_0^3 = \frac{-G_1 + \sqrt{G_1^2 + 4H_1^3}}{2}, \quad v_0^3 = \frac{-G_1 - \sqrt{G_1^2 + 4H_1^3}}{2}, \tag{3.32}$$

satisfying

$$u_0 v_0 \;=\; -H_1. \tag{3.33}$$

Thus, P is given by

$$p^2 \;=\; \delta - b_1 \tag{3.34}$$

and q and q' are then to be calculated from equations (3.23)-(3.25) and (3.34). Equation (3.22) gives

$$\left.\begin{aligned} z_1^* &= \frac{-p + \sqrt{p^2 - 4q}}{2}, \quad & z_2^* &= \frac{-p + \sqrt{p^2 - 4q}}{2}, \\ z_3^* &= \frac{p + \sqrt{p^2 - 4q'}}{2}, \quad & z_4^* &= \frac{p - \sqrt{p^2 - 4q'}}{2}, \end{aligned}\right\} \tag{3.35}$$

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as the roots of biquadratic equation (3.20). Hence the eigenvalues of the problem are $s = s_1, s_2, s_3, s_4$, where

$$s_L \;=\; \frac{Z_L^* - a_1}{a_0} \;\text{ for }\; L \;=\; 1,2,3,4, \tag{3.36}$$

which are usually distinct. The corresponding eigenvectors are

$$X_L^T \;=\; [P_L, 1] \;\text{ for }\; L \;=\; 1,2,3,4, \tag{3.37}$$

where

$$P_L = \frac{-[es_L^2 - iks_L(1+c) - k^2 d]}{as_L^2 - 2ikes_L + \hat{\omega}^2 - k^2} = \frac{-[s_L^2 - 2ikds_L + \hat{\omega}^2 - bk^2]}{es_L^2 - iks_L(1+c) - k^2 d} \;\text{ for }\; L = 1,2,3,4 \tag{3.38}$$

and $(...)^T$ denote the transpose of the matrix $(...)$. Thus, a general solution of the vector-matrix differential equation (3.14) for a monoclinic elastic medium for non-repeated eigenvalues is of the form

$$N(\alpha, k) \;=\; \sum_{L=1}^{4} (B_L X_L e^{sL^{\alpha}}), \tag{3.39}$$

where coefficients $B_1,\ B_2,\ B_3,\ B_4$ are to be determined from prescribed boundary conditions. These coefficients may depend upon $\hat{\omega}$ and k. The case of repeated eigenvalues can be tackled by making suitable modification in the expression on the right side of equation (3.39), as discussed by Ross [25]. Equation (3.39) gives the following expressions for the dimensionless displacements in the transformed domain:

$$\bar{U}(\alpha, k) \;=\; B_1 P_1 e^{s_1\alpha} + B_2 P_2 e^{s_2\alpha} + B_3 P_3 e^{s_3\alpha} + B_4 P_4 e^{s_4\alpha}, \tag{3.40}$$

$$\bar{V}(\alpha, k) \;=\; B_1 e^{s_1\alpha} + B_2 e^{s_2\alpha} + B_3 e^{s_3\alpha} + B_4 e^{s_4\alpha}. \tag{3.41}$$

Now we shall obtain the expressions for the stress components in the dimensionless form for the present plane-strain problem in the transformed domains. Equations (2.17), (2.20) and (3.1) yield

$$\sigma_{11} \;=\; \left(a\,\frac{\partial \hat{U}}{\partial \alpha} + c\,\frac{\partial \hat{V}}{\partial \beta} + e\,\frac{\partial \hat{U}}{\partial \beta} + e\,\frac{\partial \hat{V}}{\partial \alpha} \right), \tag{3.42}$$

$$\sigma_{12} \;=\; \left(e\,\frac{\partial \hat{U}}{\partial \alpha} + d\,\frac{\partial \hat{V}}{\partial \beta} + \frac{\partial \hat{U}}{\partial \beta} + \frac{\partial \hat{V}}{\partial \alpha} \right). \tag{3.43}$$

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The Fourier transformation of equations (3.42) and (3.43) yields

$$\bar{\sigma}_{11} = \left(a\frac{\partial \bar{U}}{\partial \alpha} - ikc\bar{V} - ike\bar{U} + e\frac{\partial \bar{V}}{\partial \alpha}\right), \tag{3.44}$$

$$\bar{\sigma}_{12} = \left(e\frac{\partial \bar{U}}{\partial \alpha} - ikd\bar{V} - ik\bar{U} + \frac{\partial \bar{V}}{\partial \alpha}\right). \tag{3.45}$$

Putting the value of transformed displacements $\bar{U}$ and $\bar{V}$ from equations (3.40) and (3.41) in equations (3.42) and (3.43), we find the following expressions for dimensionless stresses for plane strain deformation for a monoclinic solid.

$$\bar{\sigma}_{11} = [Q_1B_1e^{s_1\alpha} + Q_2B_2e^{s_2\alpha} + Q_3B_3e^{s_3\alpha} + Q_4B_4e^{s_4\alpha}], \tag{3.46}$$

$$\bar{\sigma}_{12} = [R_1B_1e^{s_1\alpha} + R_2B_2e^{s_2\alpha} + R_3B_3e^{s_3\alpha} + R_4B_4e^{s_4\alpha}], \tag{3.47}$$

where

$$\begin{aligned} Q_L &= aP_Ls_L - ikc - ikeP_L + s_Le, \\ R_L &= eP_Ls_L - ikd - ikP_L + s_L. \quad \text{for} \quad L = 1,2,3,4. \end{aligned} \tag{3.48}$$

From now onwards, for the sake of simplicity, we are considering more symmetric material which is a special case of monoclinic symmetry, i.e., orthotropic symmetry. For an orthotropic material, we have

$$d = 0, \quad e = 0, \tag{3.49}$$

$$\left.\begin{aligned} a_0 &= a, \quad a_1 = 0, \quad a_2 = \frac{[\hat{\omega}^2(a+1) - (ab - c^2 - 2c)k^2]}{6}, \\ a_3 &= 0, \quad a_4 = [\hat{\omega}^4 - \hat{\omega}^2(1+b)k^2 + bk^4], \end{aligned}\right\} \tag{3.50}$$

$$\left.\begin{aligned} H &= \frac{a}{6}[\hat{\omega}^2(a+1) - (ab - c^2 - 2c)k^2], \quad G = 0, \\ I &= a[\hat{\omega}^4 - \hat{\omega}^2(1+b)k^2 + bk^4] + \frac{1}{12}[\hat{\omega}^2(a+1) - (ab - c^2 - 2c)k^2]^2, \end{aligned}\right\} \tag{3.51}$$

$$p = 0, \quad q = 3H + \sqrt{12H^2 - a^2I}, \quad q' = 3H - \sqrt{12H^2 - a^2I}, \tag{3.52}$$

$$z_1 = \sqrt{-q}, \quad z_2 = -z_1, \quad z_3 = \sqrt{-q'}, \quad z_4 = -z_3, \tag{3.53}$$

$$s_1 = \frac{z_1}{a}, \quad s_2 = -s_1, \quad s_3 = \frac{z_3}{a}, \quad s_4 = -s_3, \tag{3.54}$$

$$\left.\begin{aligned} P_L &= \frac{iks_L(1+c)}{as_L^2 + \hat{\omega}^2 - k^2} = \frac{-s_L^2 - \hat{\omega}^2 + bk^2}{-iks_L(1+c)} \quad \text{for} \quad L = 1,3 \\ \text{and} \\ P_2 &= -P_1, \quad P_4 = -P_3 \end{aligned}\right\}. \tag{3.55}$$

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With real parts of $\{s_1, s_2\}$ as positive. From equations (3.40), (3.41) and (3.55), we get the following relations for an orthotropic elastic medium

$$\bar{U}(\alpha, k) \;=\; C_1 P_1 e^{s_1\alpha} - C_2 P_2 e^{-s_1\alpha} + C_3 P_3 e^{s_3\alpha} - C_4 P_3 e^{-s_3\alpha}, \tag{3.56}$$

$$\bar{V}(\alpha, k) \;=\; C_1 e^{s_1\alpha} + C_2 e^{-s_1\alpha} + C_3 e^{s_3\alpha} + C_4 e^{-s_3\alpha}, \tag{3.57}$$

where coefficients $C_1,\ C_2,\ C_3$ and C_4 are to be determined from prescribed boundary conditions. In general, for an orthotropic medium, the possibility for the presence of repeated roots in characteristic equation (3.1) is very few, so the root s_1 is different from root s_3, but if the repeated roots are present, say, roots $s_1 = s_3 = s$, then the solution of equation (3.14) is of the type Ross [25].

$$N(\alpha, k\hat{\omega}) \;=\; (C_1 + C_3\alpha) X_1^T e^{s\alpha} + (C_2 + C_4\alpha) X_2^T e^{-s\alpha}. \tag{3.58}$$

In which $C_1,\ C_2,\ C_3$ and C_4 are constant. However, in the present study, we concentrate on the general case in which $s_1 \neq s_3$. From equations (3.46), (3.47), (3.55)-(3.57) we get the following dimensionless stresses for plane strain deformation of an orthotropic medium in the transform domain:

$$\begin{aligned}\bar{\sigma}_{11} \;=\; & [(aP_1 s_1 - ikc) C_1 e^{s_1\alpha} + (aP_1 s_1 - ikc) C_2 e^{-s_1\alpha} + (aP_3 s_3 - ikc) C_3 e^{s_3\alpha} \\ & + (aP_3 s_3 - ikc) C_4 e^{-s_3\alpha}], \end{aligned} \tag{3.59a}$$

$$\begin{aligned}\bar{\sigma}_{12} \;=\; & [(-ikP_1 + s_1) C_1 e^{s_1\alpha} + (ikP_1 - s_1) C_2 e^{-s_1\alpha} + (-ikP_3 + s_3) C_3 e^{s_3\alpha} \\ & + (ikP_3 - s_3) C_4 e^{-s_3\alpha}], \end{aligned} \tag{3.59b}$$

The corresponding displacements and stresses for an isotropic elastic solid in the transform domain can be obtained from equations (3.56), (3.57), (3.59a) (3.59b) on taking

$$a \;=\; b \;=\; (\lambda + 2\mu)/\mu, \quad c \;=\; \lambda/\mu. \tag{3.60}$$

4. Application: An Impulsive Line-Load in an Infinite Orthotropic Medium

Now we consider a particular case in which harmonic time-dependent normal line-load is acting vertically downwards in the interior of an infinite orthotropic elastic medium. We shall determine the resulting displacements and stresses at any point of the medium. We take the x-axis vertically downwards and the given normal line-load $P_0 e^{i\omega t}$, acting on the z-axis (see figure 1). The elastic medium which occupies the region $x > 0$ shall be called Medium-I and the region for which $x < 0$ shall be called Medium-II. Medium-I and Medium-II have the identical elastic properties.

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Boundary Conditions: On the interface $x = 0$, the boundary conditions for the displacements are

$$U(x = 0^+, y, \omega) \;=\; U(x = 0^-, y, \omega), \tag{4.1}$$

$$V(x = 0^+, y, \omega) \;=\; V(x = 0^-, y, \omega), \tag{4.2}$$

and for stresses are

$$T_{12}(x = 0^+, y, \omega) \;=\; T_{12}(x = 0^-, y, \omega), \tag{4.3}$$

$$T_{11}(x = 0^+, y, \omega) - T_{11}(x = 0^-, y, \omega) \;=\; -P_0\delta(y). \tag{4.4}$$

Here, $\delta(y)$ is Dirac-delta function, having the following properties,

$$\int_{-\infty}^{\infty} \delta(y)\, dy \;=\; 1, \quad \delta(y) \;=\; \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-iky}\, dk, \quad F[\delta(y)] \;=\; 1. \tag{4.5}$$

We define the dimensionless force $\hat{P}_0$ as

$$\hat{P}_0 \;=\; \frac{P_0}{d_{66}}. \tag{4.6}$$

Changing equations (4.1)-(4.4) into dimensionless form and then taking Fourier transformation, we obtain

$$\bar{U}(0^+, k, \hat{\omega}) \;=\; \bar{U}(0^-, k, \hat{\omega}), \tag{4.7}$$

$$\bar{V}(0^+, k, \hat{\omega}) \;=\; \bar{V}(0^-, k, \hat{\omega}), \tag{4.8}$$

$$\bar{\sigma}_{12}(0^+, k, \hat{\omega}) \;=\; \bar{\sigma}_{12}(0^-, k, \hat{\omega}), \tag{4.9}$$

$$\bar{\sigma}_{11}(0^+, k, \hat{\omega}) - \bar{\sigma}_{11}(0^-, k, \hat{\omega}) \;= -\hat{P}_0. \tag{4.10}$$

Figure 1: Physical Model

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The coefficients C_1 and C_3 in equations (3.56), (3.57), (3.59a), (3.59b) must be zero for the transformed displacements and stresses in Medium-I otherwise, they become infinite as $\alpha \to \infty$. Therefore, transformed displacement and stress components in Medium-I, are

$$\bar{U} = [-C_2 P_1 e^{-s_1\alpha} - C_4 P_3 e^{-s_3\alpha}], \tag{4.11}$$

$$\bar{V} = [C_2 e^{-s_1\alpha} + C_4 e^{-s_3\alpha}], \tag{4.12}$$

$$\bar{\sigma}_{11} = [(aP_1 s_1 - ikc)C_2 e^{-s_1\alpha} + (aP_3 s_3 - ikc)C_4 e^{-s_3\alpha}], \tag{4.13}$$

$$\bar{\sigma}_{12} = [(ikP_1 - s_1)C_2 e^{-s_1\alpha} + (ikP_3 - s_3)C_4 e^{-s_3\alpha}]. \tag{4.14}$$

Similarly, transformed displacement and stress components in Medium-II, are

$$\bar{U} = [C_1 P_1 e^{s_1\alpha} + C_3 P_3 e^{s_3\alpha}], \tag{4.15}$$

$$\bar{V} = [C_1 e^{s_1\alpha} + C_3 e^{s_3\alpha}], \tag{4.16}$$

$$\bar{\sigma}_{11} = [(aP_1 s_1 - ikc)C_1 e^{s_1\alpha} + (aP_3 s_3 - ikc)C_3 e^{s_3\alpha}], \tag{4.17}$$

$$\bar{\sigma}_{12} = [(-ikP_1 + s_1)C_1 e^{s_1\alpha} + (-ikP_3 + s_3)C_3 e^{s_3\alpha}]. \tag{4.18}$$

Applying the boundary conditions, we get the following values of the coefficients $C_1,\ C_2,\ C_3$ and C_4 for an orthotropic elastic medium

$$C_1 = -C_2 = -C_3 = C_4 = \frac{\hat{P}_0}{2a(P_1 s_1 - P_3 s_3)}. \tag{4.19}$$

Thus, using equation (4.19), equations (4.11)-(4.18) give the displacement and stress components in the Fourier transformed domain in an infinite orthotropic elastic medium due to an impulsive line-load acting on the surface $x = 0$.

The corresponding solution for an isotropic infinite medium can be obtained from the above results by using the relation (3.60). The comparison results so obtained for an isotropic medium with the well-known results of past works is not simple, as the method to derive them is new.

To get the displacement and stress components in the frequency domain, we shall invert them numerically in the frequency domain with the help of inversion technique as discussed in the succeeding Section 5.

5. Inversion Technique

To get the function $\hat{f}(\alpha, \beta, \hat{\omega})$ in the frequency domain from $\bar{f}(\alpha, k, \beta)$, we shall make use of the relation given in equation (3.9). We know that each function can be written uniquely as the sum of an even and odd function. We write

$$\bar{f} = \bar{f}_e + \bar{f}_0, \tag{5.1}$$

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where

$$\left.\begin{aligned} \bar{f}_e(\alpha,k,\hat{\omega}) &= \frac{1}{2}\Big\{\bar{f}(\alpha,k,\hat{\omega})+\bar{f}(\alpha,-k,\hat{\omega})\Big\} \\ \bar{f}_0(\alpha,k,\hat{\omega}) &= \frac{1}{2}\Big\{\bar{f}(\alpha,k,\hat{\omega})-\bar{f}(\alpha,-k,\hat{\omega})\Big\} \end{aligned}\right\} \tag{5.2}$$

Here, $\bar{f}_e$ is an even function in k and $\bar{f}_0$ is an odd function in k.

From equations (3.9), (5.1) and (5.2), we obtain

$$\hat{f}(\alpha,\beta,\hat{\omega}) = \frac{1}{\pi}\int_0^\infty ((\cos k\beta)\bar{f}_e - i(\sin k\beta)\bar{f}_0)\,dk. \tag{5.3}$$

The method for evaluating integral in equation (5.3) is described by Press *et al.* [24], which involves the use of Romberg's integration with adaptive step size. This method also uses the results from successive refinements of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

6. Results and Discussion

In equations (4.11)-(4.19), we have obtained the dimensionless transformed displacement and stress components. The numerical inversion technique, discussed in Section 5, has been used to obtain the values of dimensionless displacement and stress components in the frequency domain. For numerical computation purposes, we use the values of elastic constants given by Love [15] for Topaz material, which are

$$a = 2.15789, \quad b = 2.67669, \quad c = 0.96240.$$

The dimensionless force $\hat{P}_0$ is taken to be of unit magnitude.

The graphs for the dimensionless elastodynamic displacement and stress components against the horizontal dimensionless distance 'β' on the plane $x = 1.0$ for various values of non-dimensional frequency, namely, $\hat{\omega}_0 = 0.2, 0.4, 0.6$ are shown through figures 2-5. Figures 2-3 represent the variation of dimensionless displacement components while figures 4 and 5 represent the variation of dimensionless stress components due to time harmonic normal line-load acting along the z-axis.

The displacement and stress components varies in the oscillatory manner due to time-harmonic dependence for all frequencies. Further, for a fixed β, the displacements and stress components increase or decrease in the oscillatory pattern as frequency $\hat{\omega}$ changes from 0.4 to 0.6.

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7. Conclusions

In this paper, we have developed a novel analytical procedure to obtain the analytical two-dimensional elastodynamic deformation of a monoclinic solid in the frequency domain. In particular, we have obtained analytical deformation in the transformed domain of an infinite orthotropic medium due to a normal line-load acting in the interior of the medium, using the numerical inversion technique as given by Press *et al.* [24]. The solution obtained in this paper for an orthotropic infinite medium in the physical domain is half-analytic. The corresponding results for an isotropic infinite medium also derivable from the above results by using the relation given by equation (3.60). The methodology developed in this paper is applicable to two-dimensional sources as well, such as strip-loading.

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