

STUDY OF SELF-SIMILAR PISTON PROBLEM IN TWO-PHASE RADIATING GASES

Kanti Pandey and Kiran Singh

Department of Mathematics and Astronomy
University of Lucknow, Lucknow (UP), India.
E-mail:- pandey_kanti@yahoo.co.in, kiransinghmth@gmail.com

Abstract: A differential approximation for the equation of radiative transfer in a grey gas is applied to study the effect of thermal radiation in case of one dimensional unsteady shock generated by a piston with velocity $v_p = ht^n$, in case of two-phase mixture of gas and dust particle. Numerical solutions in region between the shock and piston problem are presented for the case of general opacity and transparent limit. The influence of particle volume fraction for various cases are discussed and is concluded that though the general behaviour of velocity, density, pressure are unaffected, yet presence of particle volume fraction shows a rising tendency in velocity, density and pressure but a decreasing tendency in net radiative flux.

Keywords and Phrases: Shock Waves, piston problem, two-phase, radiation.

2010 Mathematics Subject Classification: Primary 76L.

1. Introduction

Effect of radiation are of great significance in astrophysical problem and nuclear explosions. On the flow field of gas, radiation effect can be expressed in terms of radiation pressure, radiation-energy density and radiation flux. In extremely high speed of flight of a space craft re-entering planetary atmosphere, radiation becomes an important mode of heat transfer and plays an important role not only in stellar-atmospheres but also in stellar-interiors. If medium is extremely rarified but extended, the energy and pressure of the radiation become comparable with those of matter and thus influences the thermodynamic properties of the medium. On account of the high temperatures that prevail in many phenomena, it is of interest to consider the effect of thermal radiation in gas-dynamics. The similarity methods of Taylor [1] and Sedov [2], well known for classical piston problems, have been extended to radiating gases by Wang [3] and Helliwell [4]. Finkleman and Baron [5] discussed the hyperbolic character of the governing equation and

applied the numerical method of characteristics to the piston problem. A number of problems relating to wave propagation with radiative effects and their extension to the magneto hydrodynamic (MHD) regime, with some consideration of non linear effects have been studied analytically by Pai [6], Vincenti and Kruger [7], Ram [8,9] and Sharma [10,11,12] at el.

By two phase flows we mean a special flow problem in which we consider the mechanism of two-phases of matter simultaneously. In general two-phase flows may be divided in two groups. The first group consists of flow problems of mixture of two-phases of four states-solids(pseudo-fluids), liquid, gas and plasma, where two phases may be mixed homogenously or inhomogenously. In second group of flow problems, interaction between the two-phases of matter through their interface is important. In present article we consider first group of two-phase flows which has its importance in internal ballistics, lunar-ash flow, exploding wire phenomena, under water explosion, astrophysics (explosion in stars), atmosphere of earth etc.

There are many engineering problems in which dilute phase of gas particle is a good approximation of actual conditions. In such cases due to the existence of solid particles in the gas, properties of mixture differ significantly from those of gas alone. Such types of studies have numerous application in underground explosions [13,14,15].

Following Wang [3] in present paper we have studied self-similar piston problem in two-phase flows of radiating gases, when ambient gas ahead of the shock is supposed to be cool with non-uniform density and the absorption coefficient is supposed to be a function of the density and temperature. The piston, which moves with a non-uniform speed, is assumed to be black and cool, so that no radiative flux enters the gas between the shock and the piston. Numerical solutions are obtained in the cases of general opacity and the transparent limit. In preparation of graphs origin 7.5 is used.

2. Basic equations

Basic equations describing a planner, cylindrically symmetric and spherically symmetric motion for two-phase flows of gas-particle mixture when radiative effects are taken into account are given by Pai [6,16].

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v) + \frac{\alpha \rho v}{x} = 0 \quad (2.1)$$

$$\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} = -\frac{1}{\rho} \frac{\partial p}{\partial x}, \quad (2.2)$$

$$\frac{\partial E}{\partial t} + v \frac{\partial E}{\partial x} - \frac{p}{\rho^2} \left(\frac{\partial \rho}{\partial t} + v \frac{\partial \rho}{\partial x} \right) = \frac{1}{\rho x^\alpha} \frac{\partial}{\partial x} (x^\alpha q_r), \quad (2.3)$$

where, $\alpha = 0, 1$ and 2 corresponds to planar, cylindrical and spherical symmetry respectively and v, p, ρ and E are resultant velocity, pressure, density and internal energy per unit mass of the mixture.

$p = p_p + p_g$, p_p and p_g being partial pressure of solid particles and gas respectively.

$p_g = (1 - Z)p$ and $p_p = Zp$, Z being the particle volume fraction,

$\rho = Z\rho_{sp} + (1 - Z)\rho_g$, ρ_{sp} and ρ_g being species density of the solid particles and partial density of gas respectively.

The internal energy per unit mass is given by

$$E = \frac{(1 - Z)p}{(\Gamma - 1)\rho} \quad (2.4)$$

where

$$Z = \frac{K_p}{(1 - K_p)G + K_p}, \quad (2.5)$$

$$G = \frac{\rho_{sp}}{\rho_g},$$

K_p being mass concentration of pseudo fluid of solid particles and is defined as

$$K_p = \frac{\bar{\rho}_p}{\rho} = \frac{Z\rho_{sp}}{\rho}. \quad (2.6)$$

In equilibrium flow K_p is a constant in the whole flow field, therefore, from equation (2.6)

$$\frac{Z}{\rho} = \text{constant}. \quad (2.7)$$

Thus equation of state becomes

$$p = \frac{(1 - K_p)}{(1 - Z)}\rho RT = \frac{\rho R_M T}{(1 - Z)}, \quad (2.8)$$

where $R_M = (1 - K_p)R$; R_M being an effective gas constant of the mixture, Ratio of specific heat is defined by,

$$\Gamma = \frac{C_{PM}}{C_{VM}} = \frac{\gamma(1 + \frac{\delta\beta}{\gamma})}{(1 + \delta\beta)} \quad (2.9)$$

$$\gamma = \frac{C_p}{C_V}, \delta = \frac{K_p}{1 - K_p}, \beta = \frac{C_{sp}}{C_V} \quad (2.10)$$

C_p and C_V being specific heat at constant pressure and constant volume and

$$C_{sp} = C_s + C_{Vp} \quad (2.11)$$

C_{Vp} being an effective specific heat at constant volume and C_s is the specific heat of the solid particles due to the internal degree of freedom. q_r the net radiative flux; determined by Traugott and Wang [17], is given by

$$q_r = q_{r+} - q_{r-} \quad (2.12)$$

$$\frac{dq_{r\pm}}{d\tau} = \pm \sqrt{3} q_{r\pm} \mp 2\pi B, \quad (2.13)$$

where q_{r+} and q_{r-} are the average radiant heat fluxes along two opposite directions in a one-dimensional space, τ is the optical thickness and B is planck's function of black body radiation; where τ and B are defined as

$$\tau = \int \theta dx \quad (2.14)$$

$$B = \frac{\sigma T^4}{\pi}, \quad (2.15)$$

where σ is Stefan's constant, T temperature and θ the absorption coefficient per unit volume. In general, θ is a complicated function of density, temperature, composition etc. For convenience it is taken here to vary according to a power law

$$\theta = \theta_1 \rho^\zeta T^\chi, \quad (2.16)$$

where θ_1, ζ, χ are constants to be determined by best fit with available opacity data. With the help of equations (2.14) to (2.16), equations (2.12) and (2.13) reduces in to following form

$$\frac{\partial q_{r+}}{\partial x} = + \frac{\theta_1}{(R')^\chi} \rho^\zeta \left(\frac{p}{q}\right)^\chi \sqrt{3} q_{r+} - \frac{2\sigma\theta_1}{(R')^{\chi+4}} \rho^\zeta \left(\frac{p}{q}\right)^{\chi+4}, \quad (2.17)$$

$$\frac{\partial q_{r-}}{\partial x} = - \frac{\theta_1}{(R')^\chi} \rho^\zeta \left(\frac{p}{q}\right)^\chi \sqrt{3} q_{r-} + \frac{2\sigma\theta_1}{(R')^{\chi+4}} \rho^\zeta \left(\frac{p}{q}\right)^{\chi+4}. \quad (2.18)$$

In the thin and thick limits (2.17, 2.18) become

$$\frac{\partial q_r}{\partial x} = - \frac{4\theta_1\sigma}{(R')^{4+\chi}} \rho^\zeta \left(\frac{p}{\rho}\right)^{\chi+4} (thin), \quad (2.19)$$

$$q_r = \frac{16\sigma}{3\theta_1} \left(R'\right)^{\chi-4} \rho^{-\zeta} \left(\frac{p}{\rho}\right)^{3-\chi} \frac{\partial}{\partial x} \left(\frac{p}{q}\right) (thick), \quad (2.20)$$

where

$$R' = \frac{R_M}{1-Z}.$$

Substituting E from (2.4) in to equation (2.3), we have

$$\frac{\partial p}{\partial t} + v \frac{\partial p}{\partial x} - a^2 \left(\frac{\partial \rho}{\partial t} + v \frac{\partial \rho}{\partial x} \right) = \frac{(\Gamma - 1)}{(1 - Z)} \left(\frac{\partial q_r}{\partial x} + \frac{\alpha q_r}{x} \right), \quad (2.21)$$

where a denotes the resulting velocity of sound given by

$$a^2 = \frac{p}{\rho(1 - Z)} \quad (2.22)$$

3. Boundary Conditions

If the subscripts **P**, **S**, 1 and 2 referred to piston, shock and the regions immediately ahead and behind the shock respectively and piston moves with velocity $v_p = ht^n$; h and n being constants, Then for the piston location,

$$x_{\mathbf{P}} = \frac{ht^{n+1}}{(n+1)},$$

to increase as t increases ($t > 0$), n must be greater than -1 . We assume that ahead of the shock, the gas is at rest and its density varies according to

$$\rho_0 = \frac{A}{x^w}, \quad (3.1)$$

where A and w are constants.

The conservation relations for the mass, momentum and energy in present problem can be written as

$$\rho_2(v_s - v_2) = \rho_1 v_s, \quad (3.2)$$

$$p_2 + \rho_2(v_s - v_2)^2 = p_1 + \rho_1 v_s^2, \quad (3.3)$$

$$E_2 + \frac{p_2}{\rho_2} + \frac{1}{2}(v_s - v_2)^2 = E_1 + \frac{p_1}{\rho_1} + \frac{1}{2}v_s^2, \quad (3.4)$$

$$\frac{Z_2}{\rho_2} = \frac{Z_1}{\rho_1}. \quad (3.5)$$

For strong shock approximation we must have

$$p_1 = T_1 = E_1 = 0.$$

From equations (3.2) to (3.5), we get

$$p_2 = \frac{2(1 - Z_1)}{\Gamma + 1} \rho_1 v_s^2, \quad (3.6)$$

$$\rho_2 = \frac{(\Gamma + 1)}{(\Gamma - 1 + 2Z_1)} \rho_1, \quad (3.7)$$

$$v_2 = \frac{2(1 - Z_1)}{(\Gamma + 1)} v_s, \quad (3.8)$$

$$(q_{r+})_2 = 0, \quad (3.9)$$

where

$$Z_1 = \frac{K_p}{(1 - K_p)G_1 + K_p}, \quad (3.10)$$

G_1 is the ratio of the density of the solid particles to the initial density of gas.

Condition (3.9) states that gas receives no radiation from infront of the shock and conditions (3.6) to (3.8) are the usual strong-shock jump conditions. It is true that the conservation of energy across the shock should contain the radiation flux term; however, since the shock is assumed to be transparent, the radiation flux is the same on the both sides, hence it can be omitted from conservation of energy equation.

However, in the thick limit as done by Elliot [18],

$q_{r1} = 0$ but $q_{r2} \neq 0$ i.e $q_{r1} \neq q_{r2}$.

The boundary conditions at the piston are

$$v = v_p = ht^n, \quad (3.11)$$

$$(q_{r-})_p = 0. \quad (3.12)$$

Again condition (3.12) states that the gas receives no radiation from the piston. For a similar flow, the shock location, x_s must be proportional to the piston location, x_p i.e. $x_p = \mu x_s$ where the constant μ is not known until the problem is solved and clearly $v_p = \mu v_s$.

4. Similarity Conditions

As suggested by Sedov [2] a self-similar flow pattern may exists in present case which reduces the governing equations into ordinary differential equations provided that exactly two independent dimensional constants arise in the complete formulation of flow problems, including boundary conditions. In the present case two independent dimensional constants h and A are introduced through boundary

conditions and radiative flux equations (2.17, 2.18) contain two new dimensional constants.

$$\frac{\theta_1}{(R')^\chi} \quad \text{and} \quad \frac{\sigma\theta_1}{(R')^{\chi+4}}$$

hence in order that similarity solution must exists, these two new constants must be expressible in terms of the dimensions of h and A . Thus following Wang [3] similarity conditions become

$$\omega = -\frac{1 + n(2\chi + 6)}{(n + 1)(1 - \zeta)} \quad (4.1)$$

and

$$\omega = \frac{n(1 + 2\chi) + 1}{(n + 1)\zeta}. \quad (4.2)$$

5. Similarity Transformation

In this section, we introduce $y = \frac{x}{x_s}$ as an independent variable such that immediately behind the shock-wave, we can choose $y = 1$ and $y = y_p$ at the piston face. Thus flow variables can be written in terms of the dimensionless function y , as given below

$$y = \frac{x}{x_s}, \quad (5.1)$$

$$\bar{v}(y) = v(x, t) \frac{t}{x}, \quad (5.2)$$

$$\bar{\rho}(y) = \frac{\rho(x, t)}{\rho_0(x)}, \quad (5.3)$$

$$\bar{p}(y) = \frac{p(x, t)}{\rho_0(x)} \frac{t^2}{x^2}, \quad (5.4)$$

$$\bar{q}_{r+}(y) = q_{r+}(x, t) \frac{t^3}{\rho_0(x)x^3} \quad (5.5)$$

$$\bar{q}_{r-}(y) = q_{r-}(x, t) \frac{t^3}{\rho_0(x)x^3} \quad (5.6)$$

Substituting equations (5.1)–(5.6) into equations (2.1), (2.2), (2.21) and (2.17, 2.18) and after certain manipulation, we have following equations

$$\bar{v}'(y) = \frac{1}{\left[1 - \frac{\Gamma \bar{p}(y)}{\bar{\rho}(y)(1 - Z)\{\bar{v}(y) - (n + 1)\}^2}\right]} \left[\frac{(\bar{v}(y) - \bar{v}^2(y))}{y\{\bar{v}(y) - (n + 1)\}} \right]$$

$$\begin{aligned}
& -\frac{\omega \bar{p}(y)}{y \bar{\rho}(y) \{\bar{v}(y) - (n+1)\}^2} + \frac{\Gamma \bar{p}(y) \bar{v}(y) (\alpha + 1)}{(1-Z) y \bar{\rho}(y) \{\bar{v}(y) - (n+1)\}^2} \\
& - \frac{(\Gamma - 1) \{(\bar{q}_{r+}(y) - \bar{q}_{r-}(y))(3 - \omega + \alpha) + y(\bar{q}'_{r+}(y) - \bar{q}'_{r-}(y))\}}{y \bar{\rho}(y) \{\bar{v}(y) - (n+1)\}^2 (1-Z)} \\
& \left. + \frac{\bar{p}(y)(\omega - 2)}{y \bar{\rho}(y) \{\bar{v}(y) - (n+1)\}} \right] \quad (5.7)
\end{aligned}$$

$$\bar{\rho}'(y) = \frac{\bar{\rho}(y)}{[\bar{v}(y) - (n+1)]} \left[\bar{v}(y) \{\omega - (\alpha + 1)\} - y \bar{v}'(y) \right], \quad (5.8)$$

$$\begin{aligned}
\bar{p}'(y) &= \frac{1}{y [\bar{v}(y) - (n+1)]} \left[\omega \bar{p}(y) - \frac{\Gamma \bar{p}(y)}{(1-Z)} \{y \bar{v}'(y) + \bar{v}(y)(\alpha + 1)\} \right. \\
& \left. + \frac{(\Gamma - 1)}{(1-Z)} \{(\bar{q}_{r+}(y) - \bar{q}_{r-}(y))(3 - \omega + \alpha) + y(\bar{q}'_{r+}(y) - \bar{q}'_{r-}(y))\} \right], \quad (5.9)
\end{aligned}$$

$$\bar{q}'_{r+}(y) = y^{-1} \left[\left(\frac{(\omega - 3) + \sqrt{3} H_1 (\bar{\rho}_1)^\zeta \left(\frac{\bar{p}}{\bar{\rho}} \right)^\chi}{y^{\frac{2\chi}{(n+1)}}} \right) \bar{q}_{r+}(y) - 2 H_2 (\bar{\rho})^\zeta \left(\frac{\bar{p}}{\bar{\rho}} \right)^{\chi+4} y^{\frac{2\chi+5}{(n+1)}} \right] \quad (5.10)$$

$$\bar{q}'_{r-}(y) = y^{-1} \left[\left(\frac{(\omega - 3) - \sqrt{3} H_1 (\bar{\rho}_1)^\zeta \left(\frac{\bar{p}}{\bar{\rho}} \right)^\chi}{y^{\frac{2\chi}{(n+1)}}} \right) \bar{q}_{r-}(y) + 2 H_2 (\bar{\rho})^\zeta \left(\frac{\bar{p}}{\bar{\rho}} \right)^{\chi+4} y^{\frac{2\chi+5}{(n+1)}} \right], \quad (5.11)$$

where the prime denotes differentiation with respect to y , and

$$H_1 = \left(\frac{\theta_1 A^\zeta}{(R')^\chi} \right) \left[\frac{h}{\mu(n+1)} \right]^{\frac{2\chi}{(n+1)}} \quad (5.12)$$

$$H_2 = \left(\frac{\sigma \theta_1 A^{\zeta-1}}{(R')^{\chi+4}} \right) \left[\frac{h}{\mu(n+1)} \right]^{\frac{2\chi+5}{(n+1)}}. \quad (5.13)$$

Substituting equations (5.1) – (5.6) into the equation (2.19), we see that for the thin limit the expressions are identical to equations (5.10, 5.11), with $H_1 = 0$. Substituting (5.1) – (5.6) into the equation (2.19) for the thick limit, we have

$$q_r = \frac{16 H_2}{3 H_1^2} \frac{y^{(5-2\chi)/(n+1)}}{\bar{\rho}^\zeta} \left[y \frac{d}{dy} \left(\frac{\bar{p}}{\bar{\rho}} \right) + \frac{2\bar{p}}{\bar{\rho}} \right]. \quad (5.14)$$

The relative magnitude of the dimensionless constants H_1 and H_2 measures the absorption property of the medium. The transformed boundary conditions at the shock ($y = 1$), are

$$\bar{\rho} = \bar{\rho}_2 = \frac{\Gamma + 1}{\Gamma - 1 + 2Z_1}, \quad (5.15)$$

$$\bar{v} = \bar{v}_2 = \frac{2(1 - Z_1)(n + 1)}{\Gamma + 1}, \quad (5.16)$$

$$\bar{p} = \bar{p}_2 = \frac{2(1 - Z_1)(n + 1)^2}{\Gamma + 1}, \quad (5.17)$$

$$\bar{q}_{r+} = (\bar{q}_{r+})_2 = 0. \quad (5.18)$$

At the piston ($y = y_p$), boundary conditions, become

$$\bar{v} = \bar{v}_p = (n + 1), \quad (5.19)$$

$$\bar{q}_{r-} = (\bar{q}_{r-})_p = 0. \quad (5.20)$$

With help of equations (3.6) to (3.10) and (5.1) to (5.6), physical variables in terms of similarity solutions are given as

$$\frac{v}{v_2} = \frac{\bar{v}(y)(\Gamma + 1)(y)}{2(1 - Z_1)(n + 1)}, \quad (5.21)$$

$$\frac{\rho}{\rho_2} = \frac{\bar{\rho}(y)(\Gamma - 1 + 2Z_1)}{\Gamma + 1} \left(\frac{1}{y^\omega} \right), \quad (5.22)$$

$$\frac{p}{p_2} = \frac{\bar{p}(y)(\Gamma + 1)}{2(1 - Z_1)(n + 1)^2} y^{2-\omega}, \quad (5.23)$$

$$\frac{q_r}{q_{r2}} = \frac{\bar{q}_r(y)}{\bar{q}_{r2}(y)} (y)^{3-\omega}. \quad (5.23)$$

The dimensionless constants, H_1, H_2 in (5.10, 5.11) depend on $h, A, \zeta, \chi, \theta_1, n, \sigma, R, \mu$ where h and A are the fundamental dimensional parameters of our problem representing separately the effects of the piston velocity and the ambient density. Constants ζ, χ, θ_1 are determined by equations (2.17, 2.18).

6. Result and Conclusion

An explicit fourth-order Runge-Kutta integration scheme is used to compute the solution of the full systems (5.7) to (5.11) for case of general radiative transfer under boundary conditions (5.15) to (5.20). The flow and field profiles are computed for following values for various case of plane, cylindrical and spherical symmetry

$$\begin{aligned}\sigma &= 5.735 \times 10^{-5} \text{erg/cm}^2 \text{sec}^0 k, R = 2.882 \times 10^{-6} \text{erg/g}^0 k, \\ K_p &= 0 \text{ and } 0.2, \theta_1 = 1.68 \times 10^{-18} \text{cm}^2/\text{g} (^0k)^5, \\ n &= -0.0625, \zeta = 1, \\ A &= 1.29 \times 10^{-4} \text{g/cm}^{3-\omega}, \omega = \frac{1}{3}, \chi = 5,\end{aligned}$$

for cases when particle mass concentration is taken into account and in absence of particle mass concentration. It is conclude that presence of particle mass concentration shows that there is an increase in velocity and density for various vales of H_2 as it is increasing. At $H_2 = 10^{13}$, presence of particle shows a drastic increase in velocity at $y = 0.96$ for spherical symmetry. Though there is an increase in density also but it is comparatively very small. As H_2 is increasing though there is a decrease in pressure but presence of particle shows increment in pressure also. The presence of particle mass concentration reduces the radiative flux in comparison to the case where particle mass concentration is not included.

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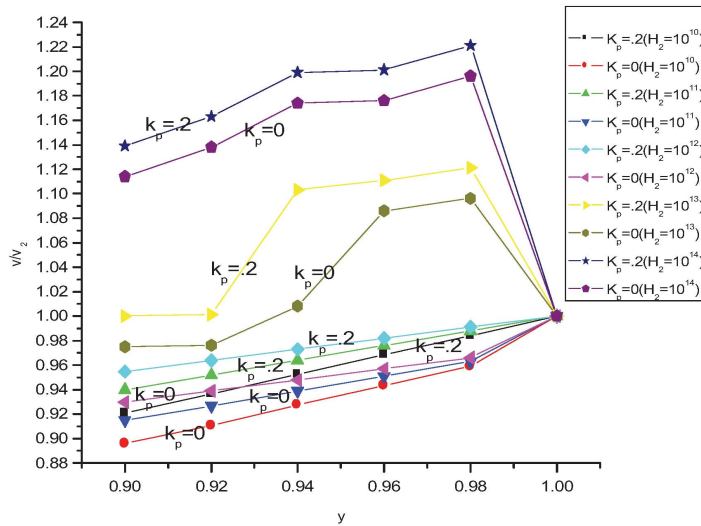
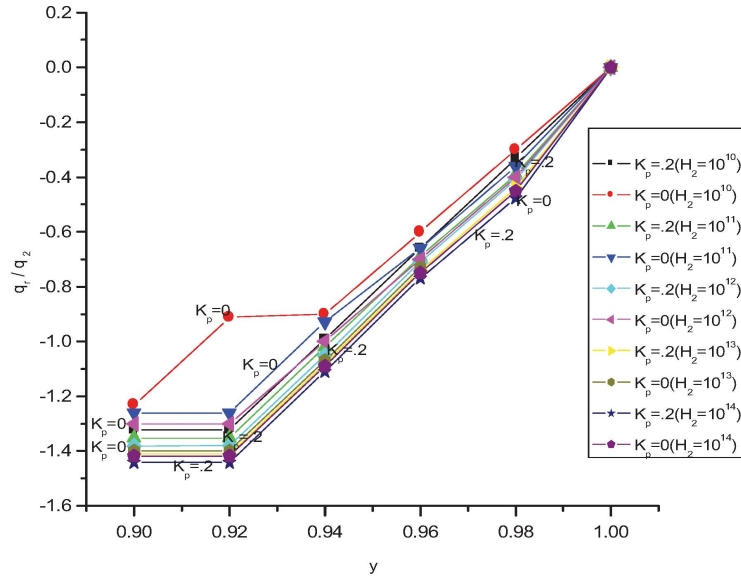
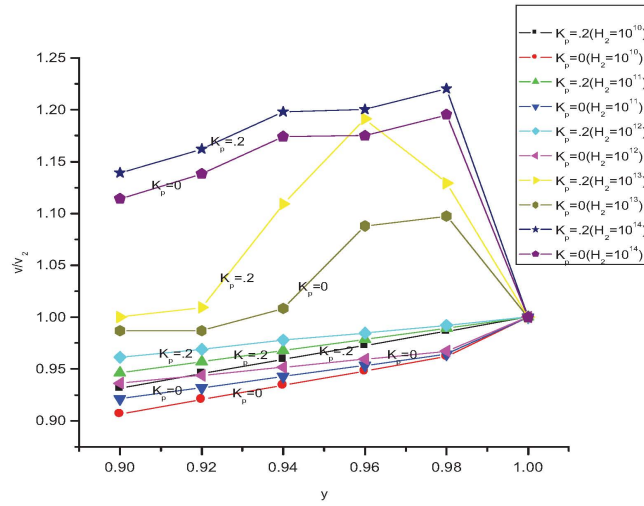
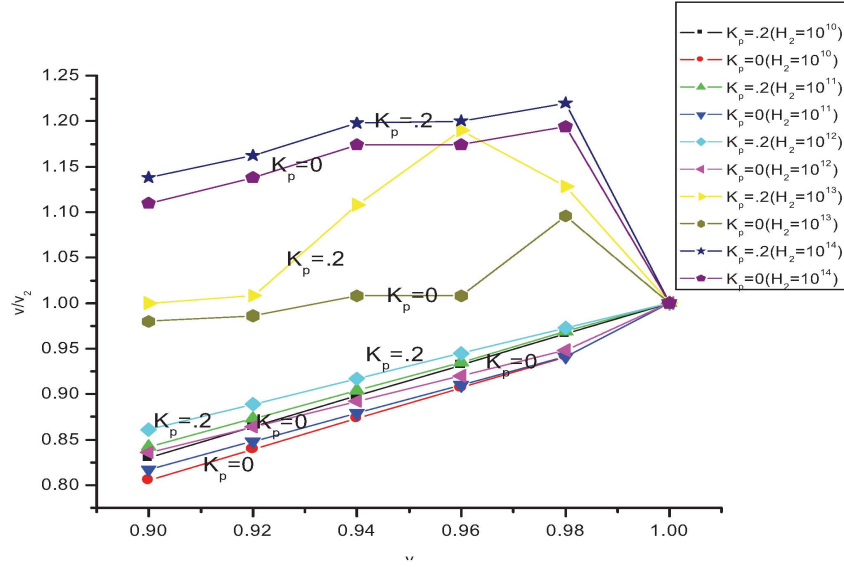
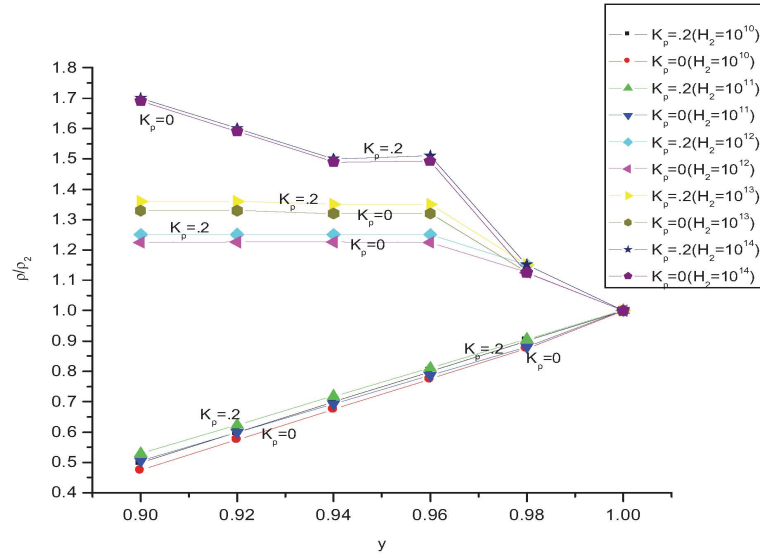


Figure 1: Variation of velocity for various values of H₂ & K_p for plane case

Figure 2: Variation of velocity for various values of H_2 and K_p for cylindrical caseFigure 3: Variation of velocity for various values of H_2 & K_p for spherical case

Figure 4: Variation of density for various values of H_2 and K_p for plane caseFigure 5: Variation of density for various values of H_2 and K_p for cylindrical case

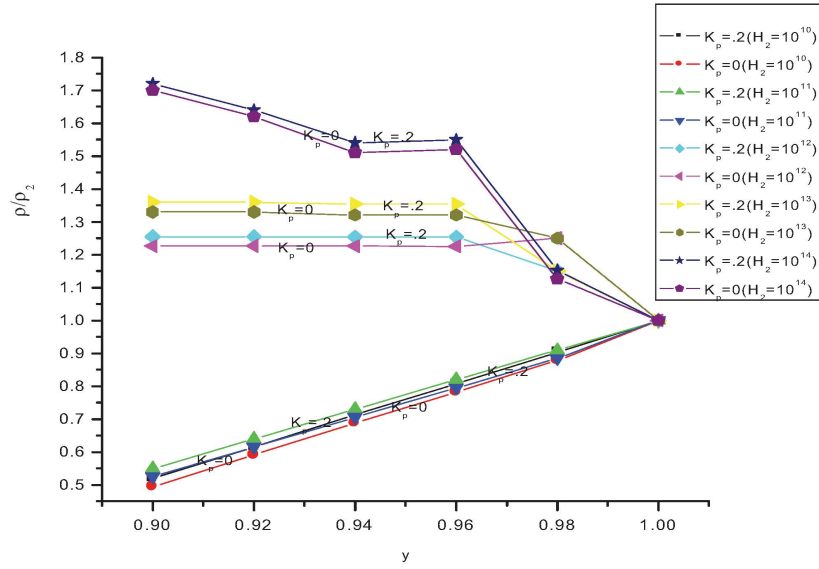


Figure 6: Variation of density for various values of H2 & Kp for spherical case

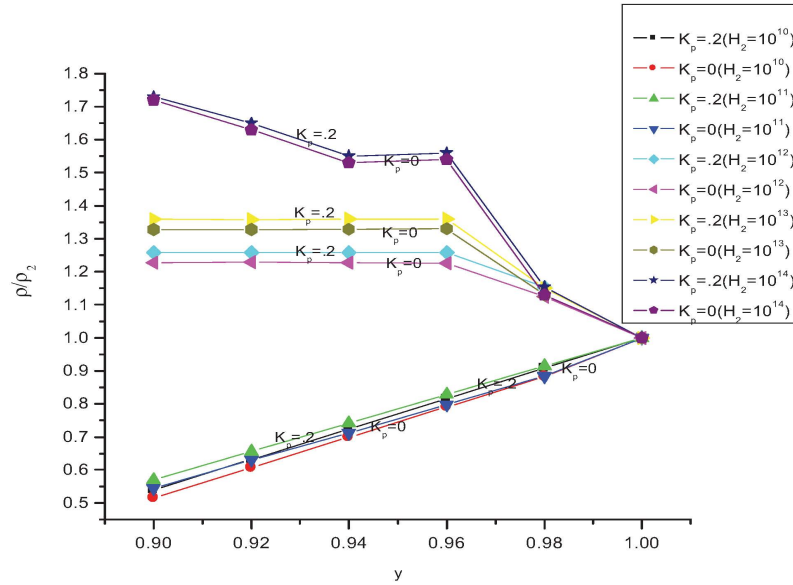
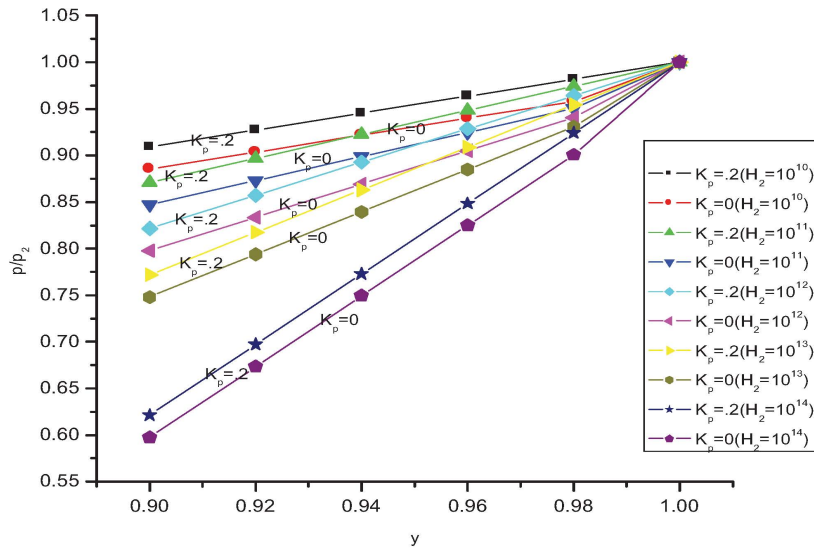
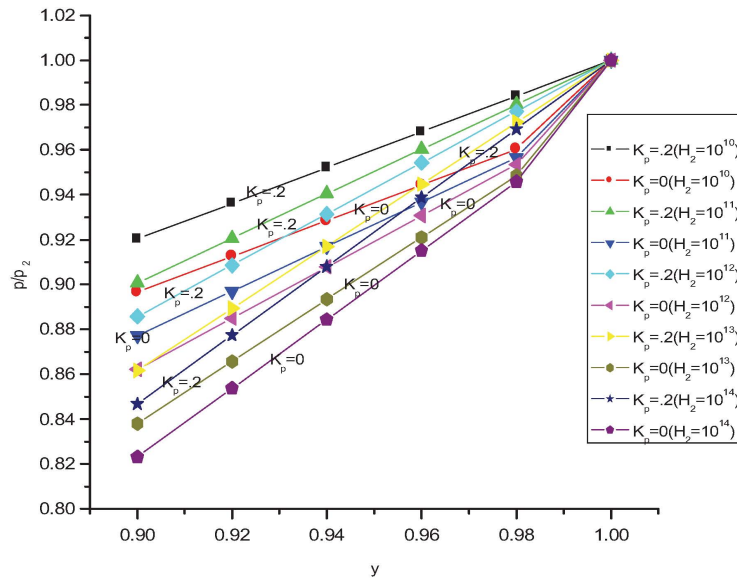


Figure 7: Variation of pressure for different values of H2 and Kp for plane case

Figure 8: Variation of pressure for various values of H_2 & K_p for cylindrical caseFigure 9: Variation of pressure for various values of H_2 & K_p for spherical case

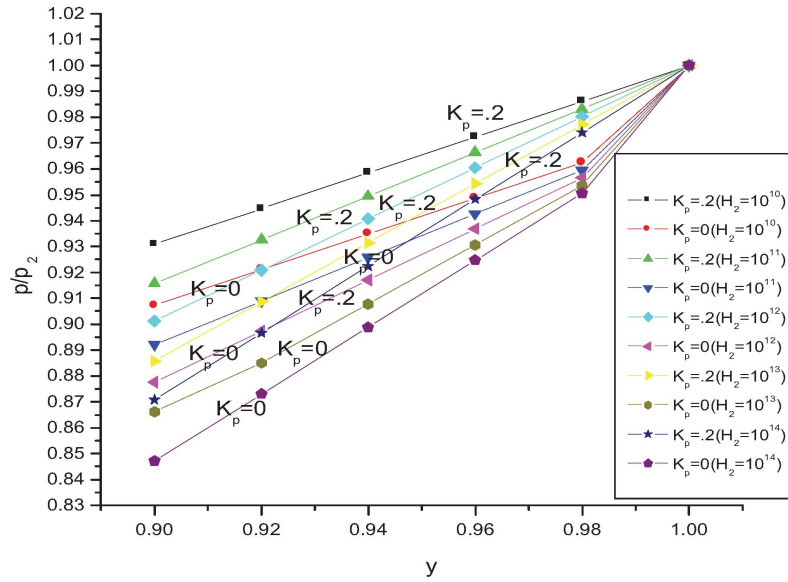


Figure 10: Variation of radiative heat flux for various values of H_2 & K_p

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