

**A NOVEL PROPERTY OF THE DICE:
NEW PALINDROMIC SEQUENCES OF NUMBERS**

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Abstract: The standard cubic dice has the pairs of numbers (1,6), (2,5) and (3,4) on its opposite faces. In this article, we extend the Avant Garde idea of one of us (KSR) to assign a place value to a permutation, to enable their ordering. As a consequence the first and subsequent differences between the place-value-ordered-permutations give rise to hierarchies of palindromic sequences. We examine the consequences of this idea to the case of standard and non-standard dice. This idea provides a reason why other pairs of numbers on the faces of the dice are not preferred, in the case of a non-standard dice. A few examples of non-standard dice are provided to establish that the new symmetry of palindromic sequences does not exist in those cases.

Keywords and Phrases: Sequences, sets, palindromic numbers.

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1. Introduction

The first non-trivial set of permutations of three numbers are:

$$(1\ 2\ 3), (2\ 3\ 1), (3\ 1\ 2), (2\ 1\ 3), (3\ 2\ 1), (1\ 3\ 2) \quad (1)$$

where, the identity $(1\ 2\ 3)$ belongs to the three cyclic permutations: $(1\ 2\ 3)$, $(2\ 3\ 1)$, $(3\ 1\ 2)$ and then the three transpositions – $(2\ 1\ 3)$, $(3\ 2\ 1)$, $(1\ 3\ 2)$. There is no rule for the ordering of the 6 permutations. A fundamental theorem states that every permutation can be written as a product of transpositions. Also the six elements when looked at from the group theoretical point of view represent the elements of C_3 , the cyclic group of three elements and all the six are the elements of the permutation group or the symmetric group of three elements of S_3 .

In an earlier paper, one of us (KSR) with Pankaj Pundir [2, 3] introduced the Avant Garde concept of assigning place values to any given permutation. This enabled:

1. Ordering the permutations given in (1) as:

$$(1\ 2\ 3), (1\ 3\ 2), (2\ 1\ 3), (2\ 3\ 1), (3\ 1\ 2), (3\ 2\ 1) \quad (2)$$

2. And the spin-off, is that the spacing between the place-valued and hence the ordered permutations give rise to the following five first differences of adjacent elements:

$$9, 81, 18, 81, 9 \quad (3)$$

which is a palindromic sequence of five numbers. The second, third, fourth and fifth differences are, respectively:

$$72, 73, 73, 72; \quad 1, 0, 1; \quad 1, 1; \quad 0 \quad (4)$$

This property was extended to establish:

3. a new property of the Symmetric group, S_n , that of ordering of the elements of the group, essentially due to place-valued permutations. It then was shown by KSR and Pankaj Pundir [3] that the differences between successive elements of this ordered, place-value assigned elements of the symmetric group S_n , give rise to a hierarchies of palindromic sequences:

$$(\mathcal{P}_k)_{1 \leq k \leq n!-1}^\beta. \quad (5)$$

In this article, we study the ordinary, standard, dice which has on its opposite faces the pairs of numbers (1,6), (2,5), and (3,4), so that the sums of each one of the three pairs is the same number 7. Why is this so? Is there a rationale behind this arrangement? These questions are studied here.

2. Standard Fair Dice

A Fair dice is one in which the probability of getting any one of the six sides face up when it is thrown randomly is equal to $1/6$. Let the six numbers on the six sides of the ordinary fair cubic dice be $(x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6)$, which for brevity of notation will be written as:

$$\left(\begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \\ 1 & 2 & 3 & 4 & 5 & 6 \end{array} \right) \equiv (1 \ 2 \ 3 \ 4 \ 5 \ 6) \quad (5)$$

In the ‘standard’ dice, the opposite sides have the pairs of numbers which add to



Figure 1: A pair of Standard Dice with (1,6), (2,5), (3,4) on opposite sides.

7 and they are the sums of the first, second and third digits of the permutation with the sixth, fifth and fourth digits, respectively. The notation for such an arrangement is:

$$(1 \ 2 \ 3 \ 4 \ 5 \ 6) = (1 \ 6) (2 \ 5) (3 \ 4) \quad (6)$$

We proceed to write down only those permutations which are such that the face sums of 7 are sacrosanct. This then gives rise to the following list of only 48 permutations out of the 720 permutations of the symmetric group S_6 , given below, in the Table, in an ascending order since the permutations have been assigned place values. Also in the third column are given the first differences between the Permutations:

$(1\ 2\ 3\ 4\ 5\ 6)$	$=$	$(1\ 6)\ (2\ 5)\ (3\ 4)$	900
$(1\ 2\ 4\ 3\ 5\ 6)$	$=$	$(1\ 6)\ (2\ 5)\ (4\ 3)$	8190
$(1\ 3\ 2\ 5\ 4\ 6)$	$=$	$(1\ 6)\ (3\ 4)\ (2\ 5)$	2700
$(1\ 3\ 5\ 2\ 4\ 6)$	$=$	$(1\ 6)\ (3\ 4)\ (5\ 2)$	7290
$(1\ 4\ 2\ 5\ 3\ 6)$	$=$	$(1\ 6)\ (4\ 3)\ (2\ 5)$	2700
$(1\ 4\ 5\ 2\ 3\ 6)$	$=$	$(1\ 6)\ (4\ 3)\ (5\ 2)$	2700
$(1\ 5\ 3\ 4\ 2\ 6)$	$=$	$(1\ 6)\ (5\ 2)\ (3\ 4)$	8190
$(1\ 5\ 4\ 3\ 2\ 6)$	$=$	$(1\ 6)\ (5\ 2)\ (4\ 3)$	900
$(2\ 1\ 3\ 4\ 6\ 5)$	$=$	$(2\ 5)\ (1\ 6)\ (3\ 4)$	59139
$(2\ 1\ 3\ 4\ 6\ 5)$	$=$	$(2\ 5)\ (1\ 6)\ (3\ 4)$	900
$(2\ 1\ 4\ 3\ 6\ 5)$	$=$	$(2\ 5)\ (1\ 6)\ (4\ 3)$	17280
$(2\ 3\ 1\ 6\ 4\ 5)$	$=$	$(2\ 5)\ (3\ 4)\ (1\ 6)$	4500
$(2\ 3\ 6\ 1\ 4\ 5)$	$=$	$(2\ 5)\ (3\ 4)\ (6\ 1)$	5490
$(2\ 4\ 1\ 6\ 3\ 5)$	$=$	$(2\ 5)\ (4\ 3)\ (1\ 6)$	4500
$(2\ 4\ 6\ 1\ 3\ 5)$	$=$	$(2\ 5)\ (4\ 3)\ (6\ 1)$	17280
$(2\ 6\ 3\ 4\ 1\ 5)$	$=$	$(2\ 5)\ (6\ 1)\ (3\ 4)$	900
$(2\ 6\ 4\ 3\ 1\ 5)$	$=$	$(2\ 5)\ (6\ 1)\ (4\ 3)$	48249
$(3\ 1\ 2\ 5\ 6\ 4)$	$=$	$(3\ 4)\ (1\ 6)\ (2\ 5)$	2700
$(3\ 1\ 5\ 2\ 6\ 4)$	$=$	$(3\ 4)\ (1\ 6)\ (5\ 2)$	6390

$(3\ 2\ 1\ 6\ 5\ 4)$	$=$	$(3\ 4)\ (2\ 5)\ (1\ 6)$	4500
$(3\ 2\ 6\ 1\ 5\ 4)$	$=$	$(3\ 4)\ (2\ 5)\ (6\ 1)$	25470
$(3\ 5\ 1\ 6\ 2\ 4)$	$=$	$(3\ 4)\ (5\ 2)\ (1\ 6)$	4500
$(3\ 5\ 6\ 1\ 2\ 4)$	$=$	$(3\ 4)\ (5\ 2)\ (6\ 1)$	6390
$(3\ 6\ 2\ 5\ 1\ 4)$	$=$	$(3\ 4)\ (6\ 1)\ (2\ 5)$	2700
$(3\ 6\ 5\ 2\ 1\ 4)$	$=$	$(3\ 4)\ (6\ 1)\ (5\ 2)$	<u>47349</u>
$(4\ 1\ 2\ 5\ 6\ 3)$	$=$	$(4\ 3)\ (1\ 6)\ (2\ 5)$	2700
$(4\ 1\ 5\ 2\ 6\ 3)$	$=$	$(4\ 3)\ (1\ 6)\ (5\ 2)$	6390
$(4\ 2\ 1\ 6\ 5\ 3)$	$=$	$(4\ 3)\ (2\ 5)\ (1\ 6)$	4500
$(4\ 2\ 6\ 1\ 5\ 3)$	$=$	$(4\ 3)\ (2\ 5)\ (6\ 1)$	25470
$(4\ 5\ 1\ 6\ 2\ 3)$	$=$	$(4\ 3)\ (5\ 2)\ (1\ 6)$	4500
$(4\ 5\ 6\ 1\ 2\ 3)$	$=$	$(4\ 3)\ (5\ 2)\ (6\ 1)$	6390
$(4\ 6\ 2\ 5\ 1\ 3)$	$=$	$(4\ 3)\ (6\ 1)\ (2\ 5)$	2700
$(4\ 6\ 5\ 2\ 1\ 3)$	$=$	$(4\ 3)\ (6\ 1)\ (5\ 2)$	48249
$(5\ 1\ 3\ 4\ 6\ 2)$	$=$	$(5\ 2)\ (1\ 6)\ (3\ 4)$	900
$(5\ 1\ 4\ 3\ 6\ 2)$	$=$	$(5\ 2)\ (1\ 6)\ (4\ 3)$	17280
$(5\ 3\ 1\ 6\ 4\ 2)$	$=$	$(5\ 2)\ (3\ 4)\ (1\ 6)$	4500
$(5\ 3\ 6\ 1\ 4\ 2)$	$=$	$(5\ 2)\ (3\ 4)\ (6\ 1)$	5490
$(5\ 4\ 1\ 6\ 3\ 2)$	$=$	$(5\ 2)\ (4\ 3)\ (1\ 6)$	4500
$(5\ 4\ 6\ 1\ 3\ 2)$	$=$	$(5\ 2)\ (4\ 3)\ (6\ 1)$	17280

$$\begin{array}{rcl}
(5 \ 6 \ 3 \ 4 \ 1 \ 2) & = & (5 \ 2) (6 \ 1) (3 \ 4) \\
& & 900 \\
(5 \ 6 \ 4 \ 3 \ 1 \ 2) & = & (5 \ 2) (6 \ 1) (4 \ 3) \\
& & 59139 \\
(6 \ 2 \ 3 \ 4 \ 5 \ 1) & = & (6 \ 1) (2 \ 5) (3 \ 4) \\
& & 900 \\
(6 \ 2 \ 4 \ 3 \ 5 \ 1) & = & (6 \ 1) (2 \ 5) (4 \ 3) \\
& & 8190 \\
(6 \ 3 \ 2 \ 5 \ 4 \ 1) & = & (6 \ 1) (3 \ 4) (2 \ 5) \\
& & 2700 \\
(6 \ 3 \ 5 \ 2 \ 4 \ 1) & = & (6 \ 1) (3 \ 4) (5 \ 2) \\
& & 7290 \\
(6 \ 4 \ 2 \ 5 \ 3 \ 1) & = & (6 \ 1) (4 \ 3) (2 \ 5) \\
& & 2700 \\
(6 \ 4 \ 5 \ 2 \ 3 \ 1) & = & (6 \ 1) (4 \ 3) (5 \ 2) \\
& & 8190 \\
(6 \ 5 \ 3 \ 4 \ 2 \ 1) & = & (6 \ 1) (5 \ 2) (3 \ 4) \\
& & 900 \\
(6 \ 5 \ 4 \ 3 \ 2 \ 1) & = & (6 \ 1) (5 \ 2) (4 \ 3)
\end{array}$$

– Seq.I (7)

Therefore, the following is the palindromic sequence established explicitly:

$$\begin{aligned}
& [900, 8190, 2700, 7290, 2700, 8190, 900, 59139, 900, 17280, 4500, 5490, 4500, \\
& 17280, 900, 48249, 2700, 6390, 4500, 25470, 4500, 6390, 2700, \underline{47349}, \\
& 2700, 6390, 4500, 25470, 4500, 6390, 2700, 48249, 900, 17280, 4500, 5490, \\
& 4500, 17280, 900, 59139, 900, 8190, 2700, 7290, 2700, 8190, 900] \quad (8)
\end{aligned}$$

In Seq. I, the element 47349 is the PIVOT for the 47 element palindromic sequence of numbers. Also, all the numbers are divisible by the common factor 9 and hence the IRREDUCIBLE sequence of palindromic numbers are (written up to only the Pivotal element, for brevity) given by:

$$\begin{aligned}
& [100, 910, 300, 810, 300, 810, 100, 6571, 100, 1920, 500, 610, 500, 1920, 100, 5361, \\
& 300, 710, 500, 2830, 500, 710, 300, \underline{5261}, \textit{palindromic}] - \textit{Seq.I} \quad (9)
\end{aligned}$$

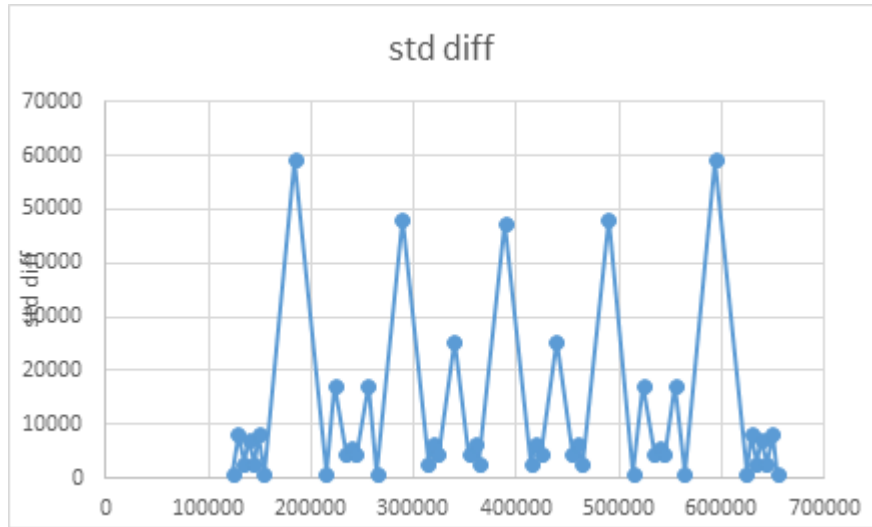


Figure 2: For a Normal Standard Dice - Place-valued permutations (123456) to (654321) plotted as a function of their first differences which gives rise a palindromic sequence.

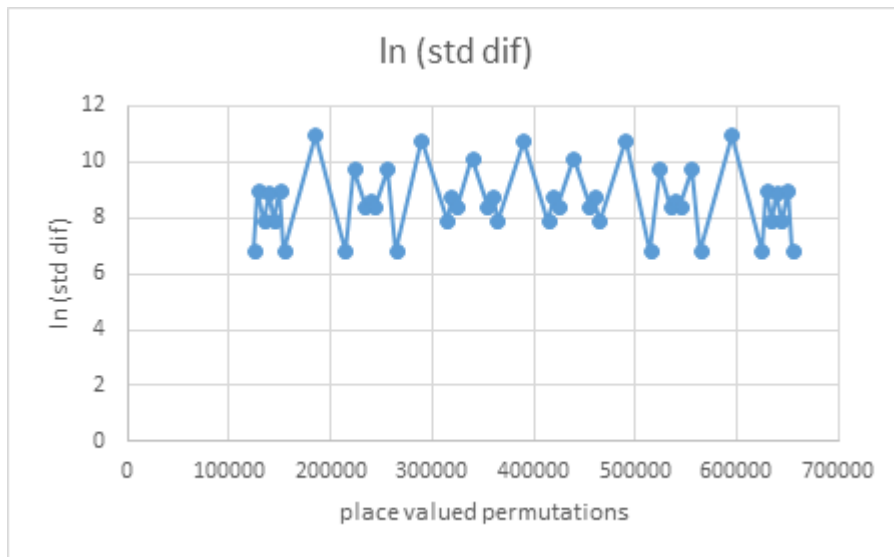


Figure 3: For a Normal Standard Dice - Place-valued permutations (123456) to (654321) plotted as a function of the logarithms of the first differences.

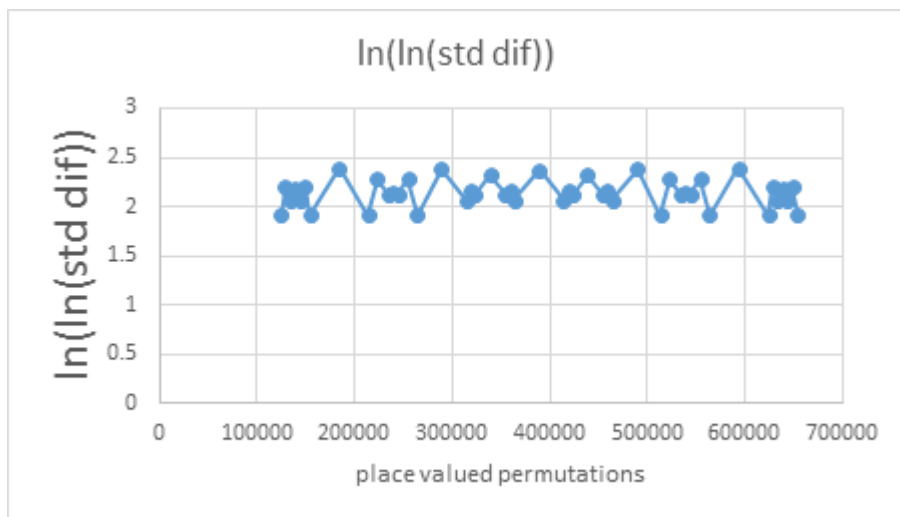


Figure 4: For a Normal Standard Dice - Place-valued permutations (123456) to (654321) plotted as a function of the log log of the first differences.

We now consider a non-standard dice, where the pairs of numbers on the opposite sides are (1,3), (2,6), (4,5), so that the opposite sides add to 4, 8, 9, instead of the same number 7 as in the case of the Fair Standard Dice, which one of us (KSR) has accidentally come across in his collection!

3. Non-Standard Fair Dice

Consider a non-standard Dice whose opposite faces are (1,3), (2,6) and (4,5), so that their sums are 4, 8, 9, instead of the 7 as in the case of a standard fair dice. Now in the notation used for the fair standard dice, a permutation for this non-standard dice is:

$$(x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6) \equiv (1 \ 2 \ 4 \ 5 \ 6 \ 3) \quad (10)$$

For this Non-Standard dice, following is the list of permutations given below, in a tabular form, in an ascending order, in the first column and in the second column are given the first differences between the adjacent Permutations:

$$\begin{array}{rcl} (1 \ 2 \ 4 \ 5 \ 6 \ 3) & = & (1 \ 3) \ (2 \ 6) \ (4 \ 5) \\ & & 900 \\ (1 \ 2 \ 5 \ 4 \ 6 \ 3) & = & (1 \ 3) \ (2 \ 6) \ (4 \ 5) \\ & & 17190 \end{array}$$



Figure 5: A pair of Non-standard Dice with (1,3), (2,6), (4,5) on opposite sides.

$(1\ 4\ 2\ 6\ 5\ 3)$	$=$	$(1\ 3)\ (4\ 5)\ (2\ 6)$	3600
$(1\ 4\ 6\ 2\ 5\ 3)$	$=$	$(1\ 3)\ (4\ 5)\ (6\ 2)$	6390
$(1\ 5\ 2\ 6\ 4\ 3)$	$=$	$(1\ 3)\ (5\ 4)\ (2\ 6)$	3600
$(1\ 5\ 6\ 2\ 4\ 3)$	$=$	$(1\ 3)\ (5\ 4)\ (6\ 2)$	8280
$(1\ 6\ 4\ 5\ 2\ 3)$	$=$	$(1\ 3)\ (6\ 2)\ (4\ 5)$	900
$(1\ 6\ 5\ 4\ 2\ 3)$	$=$	$(1\ 3)\ (6\ 2)\ (5\ 4)$	49113
$(2\ 1\ 4\ 5\ 3\ 6)$	$=$	$(2\ 6)\ (1\ 3)\ (4\ 5)$	900
$(2\ 1\ 5\ 4\ 3\ 6)$	$=$	$(2\ 6)\ (1\ 3)\ (5\ 4)$	19080
$(2\ 3\ 4\ 5\ 1\ 6)$	$=$	$(2\ 6)\ (3\ 1)\ (4\ 5)$	900
$(2\ 3\ 5\ 4\ 1\ 6)$	$=$	$(2\ 6)\ (3\ 1)\ (5\ 4)$	5940
$(2\ 4\ 1\ 3\ 5\ 6)$	$=$	$(2\ 6)\ (4\ 5)\ (1\ 3)$	1800
$(2\ 4\ 3\ 1\ 5\ 6)$	$=$	$(2\ 6)\ (4\ 5)\ (3\ 1)$	8190

$(2\ 5\ 1\ 3\ 4\ 6)$	$=$	$(2\ 6)\ (5\ 4)\ (1\ 3)$	1800
$(2\ 5\ 3\ 1\ 4\ 6)$	$=$	$(2\ 6)\ (5\ 4)\ (3\ 1)$	71415
$(3\ 2\ 4\ 5\ 6\ 1)$	$=$	$(3\ 1)\ (2\ 6)\ (4\ 5)$	900
$(3\ 2\ 5\ 4\ 6\ 1)$	$=$	$(3\ 1)\ (2\ 6)\ (5\ 4)$	17190
$(3\ 4\ 2\ 6\ 5\ 1)$	$=$	$(3\ 1)\ (4\ 5)\ (2\ 6)$	3600
$(3\ 4\ 6\ 2\ 5\ 1)$	$=$	$(3\ 1)\ (4\ 5)\ (6\ 2)$	6390
$(3\ 5\ 2\ 6\ 4\ 1)$	$=$	$(3\ 1)\ (5\ 4)\ (2\ 6)$	3600
$(3\ 5\ 6\ 2\ 4\ 1)$	$=$	$(3\ 1)\ (5\ 4)\ (6\ 2)$	8280
$(3\ 6\ 4\ 5\ 2\ 1)$	$=$	$(3\ 1)\ (6\ 2)\ (4\ 5)$	900
$(3\ 6\ 5\ 4\ 2\ 1)$	$=$	$(3\ 1)\ (6\ 2)\ (5\ 4)$	47214
$(4\ 1\ 2\ 6\ 3\ 5)$	$=$	$(2\ 6)\ (5\ 4)\ (3\ 1)$	3600
$(4\ 1\ 6\ 2\ 3\ 5)$	$=$	$(2\ 6)\ (5\ 4)\ (3\ 1)$	5130
$(4\ 2\ 1\ 3\ 6\ 5)$	$=$	$(2\ 6)\ (5\ 4)\ (3\ 1)$	1800
$(4\ 2\ 3\ 1\ 6\ 5)$	$=$	$(2\ 6)\ (5\ 4)\ (3\ 1)$	9450
$(4\ 3\ 2\ 6\ 1\ 5)$	$=$	$(2\ 6)\ (5\ 4)\ (3\ 1)$	3600
$(4\ 3\ 6\ 2\ 1\ 5)$	$=$	$(2\ 6)\ (5\ 4)\ (3\ 1)$	25110
$(4\ 6\ 1\ 3\ 2\ 5)$	$=$	$(2\ 6)\ (5\ 4)\ (3\ 1)$	1800
$(4\ 6\ 3\ 1\ 2\ 5)$	$=$	$(2\ 6)\ (5\ 4)\ (3\ 1)$	49509
$(5\ 1\ 2\ 6\ 3\ 4)$	$=$	$(5\ 4)\ (5\ 4)\ (3\ 1)$	3600
$(5\ 1\ 6\ 2\ 3\ 4)$	$=$	$(5\ 4)\ (5\ 4)\ (3\ 1)$	5130

$$\begin{array}{rcl}
(5 \ 2 \ 1 \ 3 \ 6 \ 4) & = & (5 \ 4) (5 \ 4) (3 \ 1) \\
& & 1800 \\
(5 \ 2 \ 3 \ 1 \ 6 \ 4) & = & (5 \ 4) (5 \ 4) (3 \ 1) \\
& & 9450 \\
(5 \ 3 \ 2 \ 6 \ 1 \ 4) & = & (5 \ 4) (5 \ 4) (3 \ 1) \\
& & 3600 \\
(5 \ 3 \ 6 \ 2 \ 1 \ 4) & = & (5 \ 4) (5 \ 4) (3 \ 1) \\
& & 25110 \\
(5 \ 6 \ 1 \ 3 \ 2 \ 4) & = & (5 \ 4) (5 \ 4) (3 \ 1) \\
& & 1800 \\
(5 \ 6 \ 3 \ 1 \ 2 \ 4) & = & (5 \ 4) (5 \ 4) (3 \ 1) \\
& & 51408 \\
(6 \ 1 \ 4 \ 5 \ 3 \ 2) & = & (6 \ 2) (1 \ 3) (4 \ 5) \\
& & 900 \\
(6 \ 1 \ 5 \ 4 \ 3 \ 2) & = & (6 \ 2) (1 \ 3) (5 \ 4) \\
& & 19080 \\
(6 \ 3 \ 4 \ 5 \ 1 \ 2) & = & (6 \ 2) (3 \ 1) (4 \ 5) \\
& & 900 \\
(6 \ 3 \ 5 \ 4 \ 1 \ 2) & = & (6 \ 2) (3 \ 1) (5 \ 4) \\
& & 5940 \\
(6 \ 4 \ 1 \ 3 \ 5 \ 2) & = & (6 \ 2) (4 \ 5) (1 \ 3) \\
& & 1800 \\
(6 \ 4 \ 3 \ 1 \ 5 \ 2) & = & (6 \ 2) (4 \ 5) (3 \ 1) \\
& & 8190 \\
(6 \ 5 \ 1 \ 3 \ 4 \ 2) & = & (6 \ 2) (5 \ 4) (1 \ 3) \\
& & 1800 \\
(6 \ 5 \ 3 \ 1 \ 4 \ 2) & = & (6 \ 2) (5 \ 4) (3 \ 1)
\end{array}$$

– Seq. II (10)

The sequence of first differences obtained for a non-standard dice whose sides are (1,3), (2,6), (4,5) is established explicitly to be:

$$\begin{aligned}
& [900, 17190, 3600, 6390, 3600, 8280, 900, 49113, 900, 19080, 900, 5940, 1800, \\
& 8190, 1800, 71415, 900, 17190, 3600, 6390, 3600, 8280, 900, 47214, 3600, 5130, \\
& 1800, 9450, 3600, 25110, 1800, 49509, 3600, 5130, 1800, 9450, 3600, 25110, 1800, \\
& 51408, 900, 19080, 900, 5940, 1800, 8190, 1800]
\end{aligned} \tag{11}$$

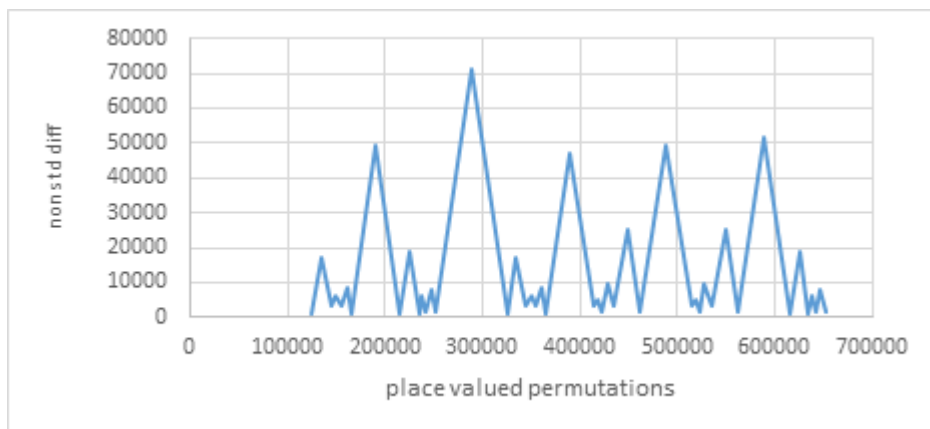


Figure 6: For a Non-Standard Dice - Place-valued permutations (123456) to (654321) plotted as a function of the first differences.

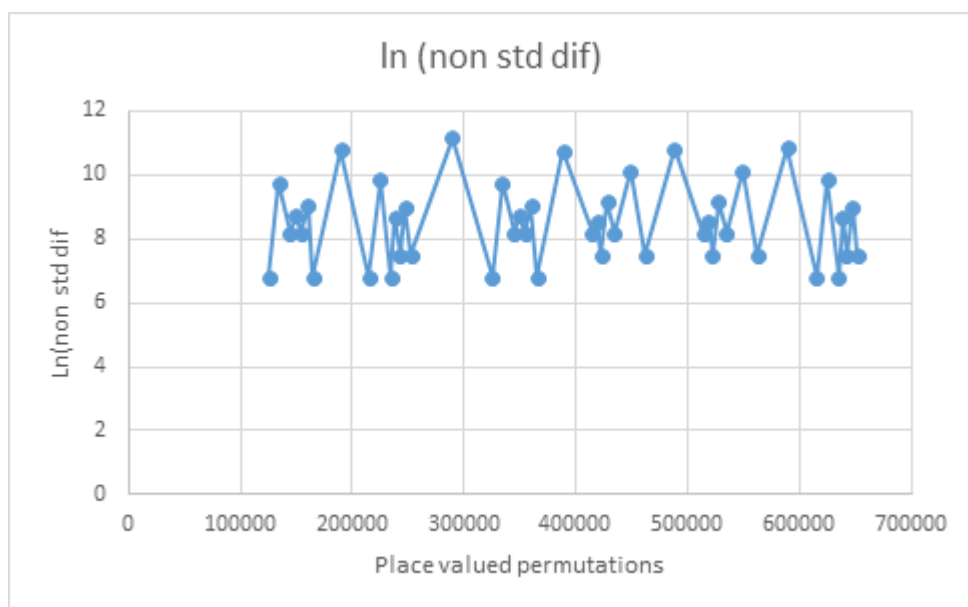


Figure 7: For a Non-Standard Dice - Place-valued permutations (123456) to (654321) plotted as a function of the logarithms of the first differences.

Consider, as another example for a Non-Standard Dice, a dice with (1,3), (2,6), (4,5) for the opposite sides of a cube. Since the idea is to establish whether or not the sequence is palindromic, it is sufficient if we show here (for brevity) only the

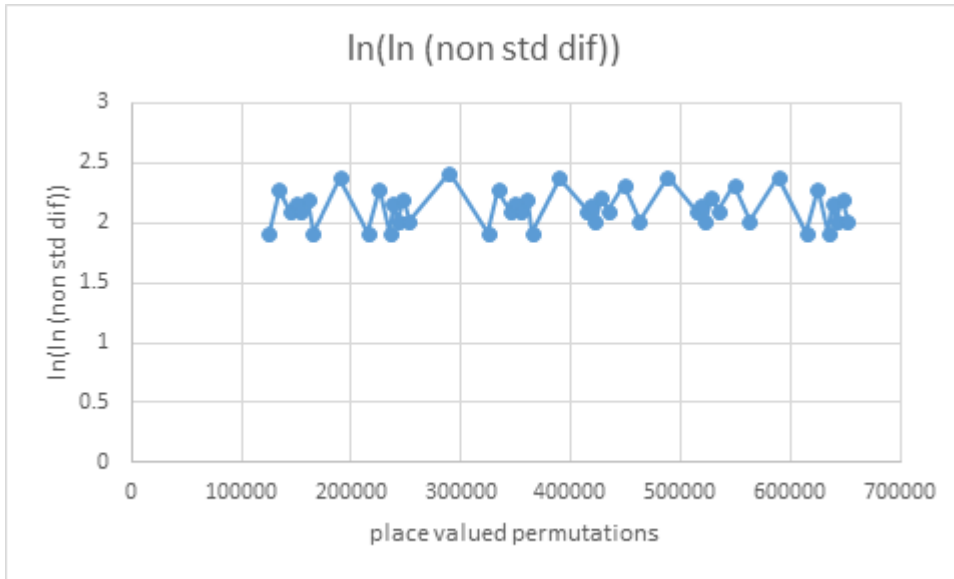


Figure 8: For a Non-Standard Dice - Place-valued permutations (123456) to (654321) plotted as a function of the log log of the first differences.

beginning and end of the sequence, which are given below:

$$\begin{array}{rcl}
 (1 \ 2 \ 5 \ 6 \ 3 \ 4) & = & (6 \ 2) (5 \ 4) (3 \ 1) \\
 & & 900 \\
 (1 \ 2 \ 6 \ 5 \ 3 \ 4) & = & (6 \ 2) (5 \ 4) (3 \ 1) \\
 & & 9090 \\
 (1 \ 3 \ 5 \ 6 \ 2 \ 4) & = & (6 \ 2) (5 \ 4) (3 \ 1) \\
 & & 900 \\
 (1 \ 3 \ 6 \ 5 \ 2 \ 4) & = & (6 \ 2) (5 \ 4) (3 \ 1) \\
 & & 15840 \\
 (1 \ 5 \ 2 \ 3 \ 4 \ 6) & = & (6 \ 2) (5 \ 4) (3 \ 1) \\
 & & 9090 \\
 (1 \ 6 \ 2 \ 3 \ 5 \ 4) & = & (6 \ 2) (5 \ 4) (3 \ 1) \\
 & & 8280 \\
 (1 \ 6 \ 4 \ 5 \ 2 \ 3) & = & (6 \ 2) (5 \ 4) (3 \ 1) \\
 & & 900 \\
 (1 \ 6 \ 5 \ 4 \ 2 \ 3) & = & (6 \ 2) (5 \ 4) (3 \ 1) \\
 \vdots & & \vdots & \vdots & \vdots
 \end{array}$$

$$\begin{array}{rcl}
(6\ 2\ 3\ 4\ 5\ 1) & = & (6\ 2)\ (5\ 4)\ (3\ 1) \\
& & 900 \\
(6\ 2\ 4\ 3\ 5\ 1) & = & (6\ 2)\ (5\ 4)\ (3\ 1) \\
& & 8190 \\
(6\ 3\ 2\ 5\ 4\ 1) & = & (6\ 2)\ (5\ 4)\ (3\ 1) \\
& & 2700 \\
(6\ 3\ 5\ 2\ 4\ 1) & = & (6\ 2)\ (5\ 4)\ (3\ 1) \\
& & 7290 \\
(6\ 4\ 2\ 5\ 3\ 1) & = & (6\ 2)\ (5\ 4)\ (3\ 1) \\
& & 2700 \\
(6\ 4\ 5\ 2\ 3\ 1) & = & (6\ 2)\ (5\ 4)\ (3\ 1) \\
& & 8190 \\
(6\ 5\ 3\ 4\ 2\ 1) & = & (6\ 2)\ (5\ 4)\ (3\ 1) \\
& & 900 \\
(6\ 5\ 4\ 3\ 2\ 1) & = & (6\ 2)\ (5\ 4)\ (3\ 1)
\end{array}$$

The Palindromic property of the sequence of 47 numbers is BROKEN! Through this counter example, it is established that anyone of the $720 - 48 = 672$ cases will not result in palindromic sequences for their first differences.

Therefore, it follows that only if the opposite sides of the fair standard dice are $(1,6)$, $(2,5)$, $(3,4)$ will this symmetric palindromic property exists. Hence, it is the preferred choice of numbering the faces of a cubic dice.

4. The case of the 4-sided Dice



Figure 9: A four sided dice

Consider the case of the 4-sided dice in which case the restricted permutations with

the property that (1234) are such that the opposite sides are (1,4) and (2,3) which is the standard case, yields for the place valued restricted partitions:

$$(1234), (1324), (2143), (2413), (3142), (3412), (4231), (4321) \quad (12)$$

yields for the first differences, the palindromic sequence:

$$[90, 819, 270, 729, 270, 819, 90] \Leftrightarrow [10, 91, 30, 81, 20, 91, 10] \quad (13)$$

where the irreducible sequence is also given.

Consider a non-standard restricted permutations (1243) where the sides are (1,3) and (2,4). In this case the allowed permutations are:

$$(1243), (1423), (2134), (2314), (3241), (3421), (4132), (4312) \quad (14)$$

and the first differences are:

$$180, 711, 180, 927, 180, 711, 180 \Leftrightarrow [20, 79, 20, 103, 20, 79, 20] \quad (15)$$

Unlike the case of the cube, in the two-dimensional case the palindromic nature of the differences exists for the standard and non-standard restricted permutations. The case of the generalized multi-sided hypercubes is under study.

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