

**A NOTE ON TIME PALINDROMIC SEQUENCES IMBEDDED IN
THE DIGITAL TIME GROUP T_G**

S. Sankaranarayanan and K. Srinivasa Rao*

Former Professor & Head, Dept. of Computer Science,
& Former Mathematics Professor, MDT Hindu College,
Thirunelveli - 627010, INDIA;
Project Consultant, Touch Lab,
Dept. of Applied Mechanics, IIT, Madras, Chennai - 600036, INDIA

*Former Senior Professor,
Institute of Mathematical Sciences, Chennai - 600113, INDIA
Former First Distinguished DST-Ramanujan Professor,
Srinivasa Ramanujan Centre, SASTRA University, Kumbakonam.
& Director (Hon),
Srinivasa Ramanujan Academy of Maths Talent, Chennai - 600020, INDIA
E-mail : ksrao18@gmail.com

(Received: Oct. 02, 2021 Accepted: Nov. 28, 2021 Published: Dec. 30, 2021)

Abstract: The digital time group T_G , defined by one of us KSR (with Pankaj Pundir) [1] has three two-digit fields for its elements: $h_2h_1 : m_2m_1 : s_2s_1$ and is identified with appropriate restricted place values on the hours (H), minutes (M) and seconds (S) fields. It is an 86,400-element Digital Time group, T_G . In this note it is shown that there exists a time-ordered sequence of 125 palindromic elements, as a subset of the group T_G , starting with 1:11:11 (interpreting, for example, 1:23:21 which without the semicolons, is 12321, a palindromic number, which reads the same when read from left to right or right to left), ending with 5:55:55 which gives rise to a new hierarchy of palindromic sequences.

Keywords and Phrases: Analytic Number Theory, sequences, palindromic sequences, digital time group, Digital Time, Finite Group, Finite Field, Order of

Elements, Palindromic sequences.

2020 Mathematics Subject Classification: 05C25, 20F65.

Recall that the elements of the Digital Time Group T_G specified by its hours, minutes, seconds fields are denoted by H:M:S or, more explicitly as

$$h_2 h_1 : m_2 m_1 : s_2 s_1$$

with

$$0 \leq h_1 \leq 3, 0 \leq h_2 \leq 2, 0 \leq m_2 \leq 5, 0 \leq m_1 \leq 9, 0 \leq s_2 \leq 5, 0 \leq s_1 \leq 9$$

for the sequence of six digits (from L to R): $h_2, h_1, m_2, m_1, s_2, s_1$.

Definition: Restrict the six digits of any element of the time group:

- $h_2 h_1$ to only h_1 (ignoring h_2)
- s_1 to h_1
- s_2 to m_2

then the elements with these restrictions of the time group will be:

$$h_1 : m_2 m_1 : m_2 h_1$$

which when interpreted as elements written ignoring the colon notation, will be

$$h_1 \ m_2 \ m_1 \ m_2 \ h_1$$

which is a palindromic number!

There are only 125 palindromic digital time group elements – starting with 1:11:11 and ending with 5:55:55 – in T_G . The first differences between adjacent time elements (in seconds) of this restricted sequence are written down with the simplifying notation that if the same time difference occurs four times, e.g. 60 60 60 60, then it will be written as:

$$(60)_4 \equiv 60 \ 60 \ 60 \ 60.$$

The adjacent time-ordered subset of 124 first-difference palindromic elements of T_G are:

$$\begin{array}{cccccccccc} (60)_4 & 370 & (60)_4 & 370 & (60)_4 & 370 & (60)_4 & 370 & (60)_4 & 921 \\ (60)_4 & 370 & (60)_4 & 370 & (60)_4 & 370 & (60)_4 & 370 & (60)_4 & 921 \end{array}$$

$$\begin{array}{cccccccccc}
(60)_4 & 370 & (60)_4 & 370 & (60)_4 & 370 & (60)_4 & 370 & (60)_4 & 921 \\
(60)_4 & 370 & (60)_4 & 370 & (60)_4 & 370 & (60)_4 & 370 & (60)_4 & 921 \\
(60)_4 & 370 & (60)_4 & 370 & (60)_4 & 370 & (60)_4 & 370 & (60)_4 &
\end{array}$$

which in the equivalent abbreviated notation is:

$$\begin{array}{cccccccccc}
((60)_4 & 370 & (60)_4 & 370 & (60)_4 & 370 & (60)_4 & 370 & (60)_4 & 921)_4 \\
(60)_4 & 370 & (60)_4 & 370 & (60)_4 & 370 & (60)_4 & 370 & (60)_4 &
\end{array}$$

where the extended notation used is for the repeated set of time elements:

$$(\underline{a \ b \ c \ \dots \ z})_n \equiv (\underline{a \ b \ c \ \dots \ z} \ \underline{a \ b \ c \ \dots \ z} \ (n \text{ times})).$$

The differences of this sequence, or, equivalently what would be the second differences of the subset of the time differences under consideration are 117 terms, given below, omitting to write down the beginning and ending zeros of the sequence¹:

$$\begin{array}{cccccccccc}
(310)_2 & (0)_3 & (310)_2 & (0)_3 & (310)_2 & (0)_3 & (310)_2 & (0)_3 & (861)_2 & \\
(0)_3 & (310)_2 & (0)_3 & (310)_2 & (0)_3 & (310)_2 & (0)_3 & (310)_2 & (0)_3 & \\
(861)_2 & (0)_3 & (310)_2 & (0)_3 & (310)_2 & (0)_3 & (310)_2 & (0)_3 & (310)_2 & \\
(0)_3 & (861)_2 & (0)_3 & (310)_2 & (0)_3 & (310)_2 & (0)_3 & (310)_2 & (0)_3 & \\
(310)_2 & (0)_3 & (861)_2 & (0)_3 & (310)_2 & (0)_3 & (310)_2 & (0)_3 & (310)_2 & \\
(0)_3 & (310)_2 & & & & & & & &
\end{array}$$

The 114 third time differences, excluding the beginning and the ending zeros are:

$$\begin{array}{cccccccccc}
310 & (0)_2 & 310 & 0 & 310 & (0)_2 & 310 & 0 & 310 & (0)_2 \\
310 & 0 & 310 & (0)_2 & 861 & 0 & 861 & (0)_2 & 310 & 0 \\
310 & (0)_2 & 310 & 0 & 310 & (0)_2 & 310 & 0 & 310 & (0)_2 \\
310 & 0 & 310 & (0)_2 & 861 & 0 & 861 & (0)_2 & 310 & 0 \\
310 & (0)_2 & 310 & 0 & 310 & (0)_2 & 310 & 0 & 310 & (0)_2 \\
310 & 0 & 310 & (0)_2 & 861 & 0 & 861 & (0)_2 & 310 & 0 \\
310 & (0)_2 & 310 & 0 & 310 & (0)_2 & 310 & 0 & 310 & (0)_2 \\
310 & 0 & 310 & (0)_2 & 861 & 0 & 861 & (0)_2 & 310 & 0 \\
310 & (0)_2 & 310 & 0 & 310 & (0)_2 & 310 & 0 & 310 & (0)_2 \\
310 & & & & & & & & &
\end{array}$$

¹The standard mathematics convention is to avoid zeros to the left of any given number, as they have no place value! Since our consideration is of palindromic sequences, naturally the zeros at the right end of any given sequence are also omitted.

The 113 fourth time differences are:

$$\begin{array}{cccccccc}
 310 & 0 & (310)_4 & 0 & (310)_4 & 0 & (310)_4 & 0 & (861)_4 \\
 0 & (310)_4 & 0 & (310)_4 & 0 & (310)_4 & 0 & (310)_4 & (861)_4 \\
 0 & (310)_4 & 0 & (310)_4 & 0 & (310)_4 & 0 & (310)_4 & (861)_4 \\
 0 & (310)_4 & 0 & (310)_4 & 0 & (310)_4 & 0 & (310)_4 & (861)_4 \\
 0 & (310)_4 & 0 & (310)_4 & 0 & (310)_4 & 0 & 310 &
 \end{array}$$

Similarly, the fifth time differences are (112 terms):

$$\begin{array}{ccccccccc}
 (310)_2 & (0)_3 & (310)_2 & (0)_3 & (310)_2 & (0)_3 & 310 & 861 & (0)_3 \\
 861 & 310 & (0)_3 & (310)_2 & (0)_3 & (310)_2 & (0)_3 & (310)_2 & (0)_3 \\
 310 & 861 & (0)_3 & 861 & 310 & (0)_3 & (310)_2 & (0)_3 & (310)_2 \\
 (0)_3 & (310)_2 & (0)_3 & 310 & 861 & (0)_3 & 861 & 310 & (0)_3 \\
 (310)_2 & (0)_3 & (310)_2 & (0)_3 & (310)_2 & (0)_3 & 310 & 861 & (0)_3 \\
 861 & 310 & (0)_3 & (310)_2 & (0)_3 & (310)_2 & (0)_3 & (310)_2 &
 \end{array}$$

In principle, it is possible to go on until the last difference is reached. It is straight forward and in the Appendix the SWIFT (Apple) program is given, which will reproduce all the results.

However, we noticed that the end of the hierarchy of sequences is reached by the 75th difference, and it is: 1 1. Recall that at each stage we have removed the beginning (and naturally the corresponding ending) zeros as already stated. So, given below are the 71st to the 76th difference. The last (76th) difference being 0. Explicitly, the 71st difference (of six time elements) to the 76th difference are:

$$\begin{array}{ccccccc}
 1 & (0)_4 & 1 & \Leftrightarrow & 1 & 0 & 0 & 0 & 0 & 1 \\
 & & & & 1 & 0 & 0 & 0 & 1 \\
 & & & & 1 & 0 & 0 & 1 \\
 & & & & 1 & 0 & 1 \\
 & & & & 1 & 1
 \end{array}$$

the 76th difference being the single element 0.

Irreducible sequences:

As in the earlier publication [1], one can define an Irreducible Sequence as one in which all the zeros are removed in the derived sequences. The following are the time-ordered irreducible palindromic sequences:

The first irreducible time differences reduce to a sequence of 49 time differences:

60 370 60 370 60 370 60 370 60 921 60 370 60 370 60 370
 60 370 60 921 60 370 60 370 60 370 60 370 60 921 60 370
 60 370 60 370 60 370 60 921 60 370 60 370 60 370 60 370 60

The second time differences reduce to a sequence of 9 irreducible time differences:

$$\mathbf{310\ 861\ 310\ 861\ 310\ 861\ 310\ 861\ 310} \equiv (\mathbf{310\ 861})_4 \mathbf{310}.$$

The third time differences reduce to a sequence of 8 irreducible differences

$$551\ 551\ 551\ 551\ 551\ 551\ 551\ 551 \Leftrightarrow (\mathbf{551})_8.$$

Concluding remarks:

In this note based on the earlier work published in this journal, we show that there exists a time-ordered sequence of 125 palindromic elements, of the group T_G , starting with 1:11:11 (interpreting it as 11111, a palindromic number) and ending with 5:55:55. The first differences give rise to a palindromic sequence of 124 elements. The second, third, fourth, etc. differences give rise to 123, 122, 121, etc. palindromic sequences. The time sequences end with the 75th difference itself. By removing the beginning and ending zeros, we get subsequences. Furthermore, by removing the intermediate zeros and retaining as a representative only one time element in the repeating elements, we obtained irreducible time sequences in this note.

Acknowledgements

One of us (KSR) wishes to thank Prof. Dr. Satya Narain Singh for a careful reading of the manuscript and another (SS) thanks Prof. Dr. M. Manivannan, Applied Mechanics Department of IITM, for his encouragement.

References

- [1] Rao K. S. and Pundir P., The Ubiquitous Digital Time Group T_G South East Asian J. of Math. & Math. Sci., Vol. 13, No. 1 (2017), 01-10.

Appendix

In this Appendix, the SWIFT 5.0 (Apple Mac) software has been used to produce the sequences referred to in the text. Program in Swift follows:

```
// Generating palindrome difference sequences without leading and
trailing zeros
// Program in Swift (Apple programming language)
// Sep 20, 2021
// By Sankaranarayanan - Guided by K S Rao
var Diff = [60, 60, 60, 60, 370, 60, 60, 60, 60, 370, 60, 60, 60,
60, 370, 60, 60, 60, 60, 370, 60, 60, 60, 60, 921, 60, 60, 60, 60,
370, 60, 60, 60, 60, 370, 60, 60, 60, 60, 370, 60, 60, 60, 60, 370,
60, 60, 60, 60, 921, 60, 60, 60, 60, 370, 60, 60, 60, 60, 370, 60,
60, 60, 370, 60, 60, 60, 60, 370, 60, 60, 60, 60, 921, 60, 60,
60, 60, 370, 60, 60, 60, 60, 370, 60, 60, 60, 60, 370, 60, 60, 60,
60, 370, 60, 60, 60, 60, 921, 60, 60, 60, 60, 370, 60, 60, 60, 60,
370, 60, 60, 60, 60, 370, 60, 60, 60, 60, 370, 60, 60, 60, 60]
var D = 0
var size = Diff.count
print("Palindromic difference sequences without leading & trailing
zeros ")
print ("-----")
print("1 Difference")
print("Size = \(size)")
print(Diff)
print("Palindromic sequence = \(Diff == Diff.reversed())")
print ("-----")
var j = 1
while size > 2 {
j = j + 1
print("\(j) Difference")
for i in 1 ..< size {
D = abs(Diff[i] - Diff[i-1])
Diff[i-1] = D
}
Diff.removeLast()
size = size - 1
// Removing leading zeros
```

```
while(Diff[0] == 0){  
  
    Diff.removeFirst()  
}  
// Removing Trailing zeros  
while(Diff[Diff.count - 1] == 0){  
    Diff.removeLast()  
}  
size = Diff.count  
print("Size = \"(size)\")  
print(Diff)  
// print("-----")  
print("Palindromic sequence = \"(Diff == Diff.reversed())\"")  
print("-----")  
}
```

